SURFACE AREA

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Word Wall

- Slant Height
- Polygon
- Lateral Face
- Vertex
- Composite Solid
- Pyramid
- Solid
- Net
- Cylinder
- Sphere
- Face
- Prism
- Cone
## Stencils
Create a set of stencils that can be used to make a set of solids.

See page 70 for details.

## Netting a Cube
Draw multiple nets for the same prism.

See page 60 for details.

## The Solar System
Find the surface area of the planets in the solar system.

See page 83 for details.

## Not Regular
What does it mean to be a regular geometric solid? Find the surface area of solids that are not regular.

See page 75 for details.

## Euler’s Formula
Discover Euler’s Formula relating the number of faces, vertices and edges in a solid.

See page 56 for details.

## A Solid Album
Make a photo album of solids.

See page 75 for details.

## Research Says...
Research a landmark or building. Calculate the surface area.

See page 79 for details.

## All the Same
Create solids that all have approximately the same surface area as a given solid.

See page 79 for details.

## Fabric Scraps
Determine how much fabric will be left over when three gifts are wrapped.

See page 70 for details.
The world is made up of many three-dimensional objects. From the blocks children play with to the Great Pyramid of Egypt, solids are all around you. A solid is a three-dimensional figure that encloses a part of space. In this lesson you will learn about five different types of solids. Many solids are made up of flat surfaces called faces. Each face is a polygon, or a closed figure made up of three or more line segments. Solids may also have one or two bases, often located on the top or bottom of a solid. The bases are either polygons or circles.

<table>
<thead>
<tr>
<th>Name</th>
<th>Definition</th>
<th>Diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prism</td>
<td>A solid formed by polygons with two congruent, parallel bases.</td>
<td><img src="image1" alt="Prism Diagram" /></td>
</tr>
<tr>
<td>Pyramid</td>
<td>A solid with a polygonal base and triangular sides that meet at a vertex.</td>
<td><img src="image2" alt="Pyramid Diagram" /></td>
</tr>
<tr>
<td>Cylinder</td>
<td>A solid formed by two congruent and parallel circular bases.</td>
<td><img src="image3" alt="Cylinder Diagram" /></td>
</tr>
<tr>
<td>Cone</td>
<td>A solid formed by one circular base and a curved surface which connects the base and the vertex.</td>
<td><img src="image4" alt="Cone Diagram" /></td>
</tr>
<tr>
<td>Sphere</td>
<td>A solid formed by a set of points in space that are the same distance from a center point.</td>
<td><img src="image5" alt="Sphere Diagram" /></td>
</tr>
</tbody>
</table>
Prisms and pyramids are named by the shape of their base(s). Each prism or pyramid has two parts to its name. The first part of the solid’s name describes the shape of the base. The second part of its name describes the shape as either a prism or a pyramid.

**Naming a Prism or Pyramid**

1. Describe the shape of the solid’s base(s).
   - 3 sides = Triangular
   - 4 sides = Square, Rectangular or Trapezoidal
   - 5 sides = Pentagonal
   - 6 sides = Hexagonal
   - 7 sides = Heptagonal
   - 8 sides = Octagonal
2. Describe the shape as either a prism or pyramid.

<table>
<thead>
<tr>
<th>Pentagonal prism</th>
<th>Triangular prism</th>
<th>Hexagonal pyramid</th>
<th>Square pyramid</th>
<th>Rectangular prism</th>
</tr>
</thead>
</table>

**Example 1**

Name the solid that best describes each picture.

a. The bases are the polygons parallel to one another. The bases are triangles. The solid is called a **triangular prism**.

b. The solid has one base with six sides. It is a **hexagonal pyramid**.

c. The solid has parallel circular bases. It is a **cylinder**.

d. The prism has parallel bases that are squares or rectangles. It can be called a **square prism** or **rectangular prism**.

There are two types of faces on a solid: bases and lateral faces. A base is usually the top or bottom of a solid. A **lateral face** is any side of a solid which is a polygon. An **edge** of a solid is a line segment where two faces meet. A **vertex** is a point where three or more edges meet. Examples of faces, vertices and edges are shown below.
EXAMPLE 2

Identify the number of faces, vertices, edges, bases and lateral faces in the hexagonal pyramid.

**Solutions**

**Bases:** The polygon at the bottom of the prism. There is 1 base.
**Lateral Faces:** All polygonal sides that are not bases. There are 6 lateral faces.
**Edges:** The line segments formed when two faces meet. There are 12 edges.
**Vertices:** All corners where the edges meet. There are 7 vertices.
**Faces:** All polygonal sides including the base and all lateral faces. There are 7 faces.

**EXERCISES**

Name the solid that best describes each picture.

1.  
2.  
3.  
4.  
5.  
6.  
7.  
8.  

Tell whether each statement is true or false. If false, explain why.

10. A cone has two bases.  
11. A cylinder is more like a prism than a pyramid.
12. A prism has two bases that are not congruent.  
13. The faces of a pyramid are rectangles.
14. The faces of a prism are rectangles.  
15. A cone has one vertex.
16. A pyramid has one vertex.  
17. All prisms have 12 vertices.
18. A prism has one base.  
19. A pyramid is named by the shape of its lateral faces.
Identify the number of lateral faces, bases, edges and vertices.

20. (Diagram of a prism)

21. (Diagram of a pyramid)

22. (Diagram of a cone)

23. Complete the table.

<table>
<thead>
<tr>
<th>Name of solid</th>
<th>a.</th>
<th>e.</th>
<th>i.</th>
<th>m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of faces (include base)</td>
<td>b.</td>
<td>f.</td>
<td>j.</td>
<td>n.</td>
</tr>
<tr>
<td>Number of vertices</td>
<td>c.</td>
<td>g.</td>
<td>k.</td>
<td>o.</td>
</tr>
<tr>
<td>Number of edges</td>
<td>d.</td>
<td>h.</td>
<td>l.</td>
<td>p.</td>
</tr>
</tbody>
</table>

24. Describe how to determine which faces are lateral faces and which faces are bases on a prism or pyramid.

25. How are the number of bases and lateral faces related to the total number of faces on a prism?

26. Explain why cylinders and cones do not have faces and edges.

Name a solid that fits each description.

27. a can of soda pop

28. a shoe box

29. a pyramid with four faces

30. a prism with eight lateral faces

31. a pyramid with six vertices

32. a prism with nine edges

33. a prism with bases that are trapezoids

34. a pyramid with seven lateral faces

35. a solid with eight faces

36. a basketball
**LESSON 10 ~ THREE-DIMENSIONAL FIGURES**

**REVIEW**

Find the circumference and area of each circle. Choose the most appropriate value of $\pi$ to use.

37. 38. 39.

40. Clara and Beatrice made a circular baby blanket. It has a radius of 24 inches. Find the area of the baby blanket. Use 3.14 for $\pi$.

---

**Tic-Tac-Toe ~ Euler’s Formula**

Leonard Euler (1707-1783) proved many theorems and developed many formulas in his life. One formula he developed relates the number of edges, faces and vertices of a solid consisting of faces that are polygons (called polyhedra).

**Step 1:** Copy and complete the table below.

<table>
<thead>
<tr>
<th>Solid</th>
<th>Vertices (V)</th>
<th>Faces (F)</th>
<th>Edges (E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Square Pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pentagonal Prism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hexagonal Pyramid</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 2:** Add a column to the table titled $V + F - E$. Find the value of $V + F - E$ for each solid.

**Step 3:** Did you notice anything about your results in **Step 2**? Can you make any predictions about the formula Euler developed?

**Step 4:** Look up Euler’s formula for polyhedra on the internet or another resource. Were you correct? State his formula.

**Step 5:** Will the formula hold true if you cut off the corner of a prism like the solid below? Show all work.
A **net** is a two-dimensional pattern that can be folded to form a three-dimensional figure. Below are some nets of prisms.

**EXPLORE!**

Locate a prism to use in this activity.

**Step 1:** Trace the base of your solid near the edge of a large piece of paper.

**Step 2:** Tip your solid down on its side. Line up the solid so that the edge of the lateral face touches the corresponding edge on the drawing. Trace this lateral face.

**Step 3:** Roll the prism on the piece of paper and trace each lateral side one at a time. Finally, stand the prism up and trace the remaining base. You should never lift the prism.

**Step 4:** Label each face as top, bottom or side. There will be more than one side. This is a net of your prism.

**Step 5:** Cut out your net. Fold it on the lines and tape it together. Does it match the figure you started with?

**Step 6:** Switch prisms with a classmate. Draw a net of your new prism. Will your net be exactly the same as your classmate’s net for the same object? Explain how the nets could be different yet still form the same solid.

**Step 7:** Look at the nets at the beginning of the lesson. What solid will each net form?
**Example 1**

Sketch a net for a triangular prism.

**Solution**

Step 1: First sketch one base.

Step 2: Sketch one lateral face that connects to the base.

Step 3: Sketch the other base.

Step 4: Finally, sketch the remaining lateral faces.

---

**Example 2**

Draw a net for a pentagonal pyramid.

**Solution**

Start by sketching the base of the pyramid (a pentagon).

Sketch each lateral face (triangle) by attaching it to one of the base edges.

Drawing solids is a difficult skill that requires attention to detail. Solids are three-dimensional. This means some edges in a solid may be hidden. Dashed segments are used for hidden edges.

---

**Drawing a Prism**

1. Draw one base.

2. Draw the second base directly above or below the first base.

3. Draw the edges using solid or dashed lines depending on whether or not each edge is hidden.

**Drawing a Pyramid**

1. Draw the base.

2. Draw a point centered above the base.

3. Connect the vertices of the square to the point. Determine whether each edge should be solid or dashed.
Drawing cylinders and cones requires using ovals rather than circles for the bases. Draw one or two ovals for the bases of a solid. Connect the ‘ends’ of each oval to the other base or a vertex. Change any solid lines to dashed lines if they are hidden.

**EXERCISES**

Sketch a net of each solid.

1. 

2. 

3. 

4. 

5. 

6. 

7. 

8. 

Sketch a diagram of each solid.

10. triangular prism  
11. square prism  
12. pentagonal prism  
13. triangular pyramid  
14. square pyramid  
15. pentagonal pyramid  
16. cone  
17. cylinder  
18. trapezoidal prism  
19. Explain how to draw a hexagonal pyramid.
20. Sketch two different real-world three-dimensional objects. Explain what real-world object each sketch represents.
 Identify the number of vertices, edges, lateral faces and bases in each solid.

21.  

22.  

23.  

Find the area of each composite figure.

24.  

25.  

**Tic-Tac-Toe ~ Netting A Cube**

A cube is a specific type of rectangular prism. The faces of a cube are all congruent squares. Create a minimum of 10 different nets that form the same cube.

1. Draw each net.

2. Cut out each net.

3. Fold to make sure it forms a cube

4. Present all nets with a creative, visual display.
The surface area of a solid is the sum of the areas of all the surfaces. This includes the areas of the bases and faces of the solid. The surface area of a prism is the sum of the area of the two bases and the area of the lateral faces.

**EXPLORE!**

**Step 1:** Sketch the net of the rectangular prism at the right.

**Step 2:** Label the edges of each face of the net with the correct lengths.

**Step 3:** Are there any rectangles that are the same shape and size? If so, how many are there of each size?

**Step 4:** Calculate the area of each face of the prism.

**Step 5:** Find the sum of the face areas. This sum is the surface area of the prism.

**Step 6:** Surface area can also be determined without drawing a net. Find the area of the bottom base of the prism used in **Step 1**.

**Step 7:** What is the area of the top of the prism?

**Step 8:** Find the perimeter of the base. Multiply the perimeter by the height of the prism.

**Step 9:** Add the answers from **Steps 6-8**. This sum is the surface area of the prism.

**Step 10:** What do you notice about the answers in **Step 9 and Step 5**? Discuss what you observed with a classmate.

**Step 11:** The Oregon State Assessment Formula Sheet lists the formula $2(lw + wh + lh)$ for the surface area of a rectangular prism.

- **a.** Use this formula to find the surface area of the prism above.
- **b.** Explain why this formula works.

**Step 12:** Find the surface area of each prism. Try at least two different methods from this Explore!

- **a.**
- **b.**
- **c.**
The lateral surface area is the sum of the areas of the lateral faces. The total surface area includes the area of the two bases.

### Lateral Surface Area of a Prism

The lateral area (LA) of a prism is equal to the perimeter ($P$) of the base times the height ($h$) of the prism.

$$LA = Ph$$

### Total Surface Area of a Prism

The surface area of a prism is equal to the sum of the lateral area (LA) and the area of the two bases (B).

$$SA = LA + 2B$$

$$SA = Ph + 2B$$

---

**Example 1**

The dimensions of a shipping container are given. Find the surface area of the container.

![Image of a shipping container with dimensions 4 ft, 2 ft, and 3 ft]

**Solution**

- Find the perimeter of the base.  
  $$4 + 2 + 4 + 2 = 12$$
- Locate the height of the prism.  
  $$h = 3$$
- Find the area of the base.  
  $$4(2) = 8$$
- Use the surface area formula for a prism.  
  $$SA = Ph + 2B$$
- Substitute all known values for the variables.  
  $$SA = 12(3) + 2(8)$$
- Multiply, then add.  
  $$SA = 36 + 16 = 52$$

The surface area of the shipping crate is 52 square feet.

---

**Example 2**

The surface area of a pentagonal prism is 1,445 square centimeters. The lateral area of the prism is 1,100 square centimeters. What is the area of one base?

**Solution**

- Use the basic surface area formula for a prism.  
  $$SA = LA + 2B$$
- Substitute all known values for the variables.  
  $$1445 = 1100 + 2B$$
- Isolate the area of the bases.  
  $$-1100 \quad -1100$$
  $$345 \quad = \frac{2B}{2}$$
  $$172.5 = B = \text{base area}$$

The area of one base in the pentagonal prism is 172.5 cm².
EXERCISES

1. a. Draw a net of the prism at the left. Label the lengths of each side.
   b. Find the area of each polygon in the net. Write the areas in the corresponding polygons.
   c. Add all areas to find the total surface area.

2. a. Draw a net of the prism at the left. Label the key information on each polygon.
   b. Find the area of each polygon in the net. Write the areas in the corresponding polygons.
   c. Add all areas to find the total surface area.

Find the surface area of each solid.

3. Base area = 60 $u^2$
   Perimeter of base = 30 units

4. Area of the base = 184 $cm^2$
   Each side of the base is 8.4 cm

5.

6. 18 units

7.

8. 5 m

9. The area of one base on a shoe box is 120 $in^2$. The lateral area is 220 $in^2$. Find the surface area of the shoe box.

10. The lateral area of an octagonal prism is 192 square meters. The area of one base is 43 square meters. What is the surface area of the octagonal prism?

11. The area of one base of a triangular prism is 18 square feet. The perimeter of the triangular base is 19 feet. The height of the prism is 21 feet. Find the surface area of the triangular prism.
12. An octagon with an area of 120 cm² is the base of a prism. Each side of the octagon is 5 cm. The height of the prism is 11 cm. Find the surface area of the octagonal prism.

13. The surface area of a triangular prism is 72 ft². The lateral area is 60 ft². Find the area of one base.

14. A cargo box has these dimensions: 5 m by 6 m by 3.2 m. Find the surface area of the box using the formula from the Oregon State Assessment Formula Sheet: $2(lw + wh + lh)$. 

15. Sergio wrapped a gift for his mother. The box was a rectangular prism with dimensions of 12 inches by 10 inches by 4 inches.
   a. How much wrapping paper did Sergio need to exactly cover the box?
   b. Sergio ended up using 450 square inches of wrapping paper. Why do you think he used more than the answer in part a?

16. The surface area of a rectangular prism is 120 square inches. The lateral area is 90 square inches.
   a. What is the area of one base?
   b. Identify a possible set of dimensions for the base of the prism.
   c. Find the height of the prism.
   d. Draw a diagram of the prism. Label the length, width and height.

17. A rectangular prism has a surface area of 846 square feet. Its height is 12 feet and its width is 9 feet. Use the formula $2(lw + wh + lh)$ to find the length of the prism.

**REVIEW**

Find the area of the shaded regions.

18.  

19.  

20.  

21.  

22.  

23. Use $\frac{22}{7}$ for $\pi$. 

---

Lesson 12 – Surface Area Of Prisms
The net of a cylinder is shown below. A cylinder’s net is made of two congruent circles and a rectangle.

EXPLORE!

Locate a cylinder to use for this activity.

Step 1: Trace the base of the cylinder near the bottom of a blank sheet of paper. Measure the radius of the cylinder’s base to the nearest tenth of a centimeter. Record the length on your diagram.

Step 2: Measure the distance around the cylinder using a tape measure or piece of string. Measure the height of the cylinder. Draw a rectangle directly above your circle. The rectangle should have a base length equal to the distance around the cylinder. The height should be equal to the height of the cylinder. Record the lengths on the diagram inside the rectangle.

Step 3: The net needs another circle congruent to the circle drawn in Step 1. Where should the circle be placed? Draw the circle on the diagram and compare with a classmate.

Step 4: Cut out the net. Wrap it around the cylinder. Did it matter where the circles were placed on the top and bottom of the rectangle?

Step 5: Find the area of each figure in the net. Use 3.14 for π. What is the total surface area of the cylinder?
Step 6: Find the circumference of the base of the cylinder. Use 3.14 for \( \pi \). How does the circumference compare to the length of the rectangle drawn in Step 2?

Step 7: If the radius and height of a cylinder are given, the surface area of the cylinder can be calculated. Write a formula using the variables for radius and height that will help find the surface area of any cylinder.

Step 8: Calculate the surface area of each cylinder. Use 3.14 for \( \pi \).

**EXAMPLE 1**

Find the surface area of the cylinder. Use 3.14 for \( \pi \).

**Solution**

Use the surface area formula.

Substitute all known values for the variables.

Multiply, then add.

The surface area of the cylinder is about 207.24 square inches.
The label on a soup can has a lateral area of approximately 395.6 square centimeters. The height of the can is 8 centimeters.

a. Find the radius of the base of the can.
b. How many square centimeters of aluminum are needed for the surface of the entire can?

**Solutions**

a. Use the formula for lateral area of a cylinder.  
\[ LA = 2\pi rh \]
Substitute all known values for the variables.  
\[ 395.6 \approx 2(3.14)(r)(9) \]
Multiply.  
\[ 395.6 \approx 56.52r \]
Divide.  
\[ \frac{395.6}{56.52} = \frac{56.52r}{56.52} \]
7 ≈ r

The radius of the base of the soup can is about 7 centimeters.

b. Use the surface area formula for a cylinder.  
\[ SA = 2\pi rh + 2\pi r^2 \]
Substitute all known values for the variables.  
\[ \approx (3.14)(7)(9) + 2(3.14)(7)^2 \]
Multiply, then add.  
\[ \approx 395.64 + 307.72 \approx 703.36 \]

The amount of aluminum needed to cover the cylinder is about 703.36 square centimeters.

**Exercises**

1. Identify 4 cylinders you use or see in real-world situations.

2. Draw a net of a cylinder with a radius of 2 cm and a height of 7 cm. Label the radius and height on the net.

3. Sketch a net of the oil drum shown at the right. It has a diameter of 3 feet and a height of 5 feet.
   a. Label the radius and height on the net.
   b. Find the length of the rectangle in the net. Label on the diagram. Use 3.14 for \( \pi \).
   c. Find the area of each figure in the net. Round to the nearest hundredth.
   d. Find the surface area of the oil drum by adding the areas of all three figures in the net.

Find the surface area of each cylinder.

4. The area of one base of a cylinder is 8 cm². The lateral area of the cylinder is 45 cm².

5. The area of one base of a cylinder is 3.5 cm². The lateral area is 8.4 cm².

6. The lateral area of a cylinder is 150 ft². The radius of the cylinder is 10 ft.
7. Use the net shown at the right.
   a. What color in the diagram represents the lateral area?
   b. Find the area of each figure in the diagram.
   c. Add the three areas together to find the total surface area.
   d. Draw a three-dimensional cylinder with the same dimensions as the net.
   e. What does the length of 6.28 represent on the three-dimensional diagram?

Find the surface area of each cylinder. Use 3.14 for \( \pi \).

8. 
   ![Diagram of a cylinder with dimensions: radius 8 ft, height 20 ft]

9. 
   ![Diagram of a cylinder with dimensions: radius 4 in, height 12 in]

10. 
    ![Diagram of a cylinder with dimensions: radius 7.5 cm, height 3.5 cm]

11. Find the surface area of a cylinder with a radius of 8 feet and a height of 15 feet.

12. A cylindrical glass is shown at the right. The height of the cup is 7 inches. The radius of the base is 1.5 inches.
    a. How many bases are included in the surface area of the outside of the glass?
    b. Find the total surface area of the outside of the glass.

13. A can of tuna fish has a diameter of 8 cm and a height of 4 cm. A label is wrapped around the outside of the can.
    a. Find the area of the label.
    b. What part of the surface area was found in part a?

14. A can of soup has a diameter of 2 inches and a height of 5 inches. Find the area of the paper label wrapped around the outside of the can.

15. A pillar candle has a radius of 1 inch and is 6 inches tall.
    a. Find the surface area of the candle.
    b. After burning for three hours, the candle is only 2 inches tall. What is the surface area of the candle now?

16. The lateral area of a cylinder is 1,318.8 square meters. The height of the cylinder is 21 meters.
    a. Find the radius of the cylinder.
    b. Find the surface area of the cylinder.

17. A semi-truck traveling from Tillamook to McMinnville carries a stainless steel container full of milk. Find the surface area of the container if the diameter of the tank is 8 feet and it is 48 feet long.
18. Doris makes miniature chocolate desserts. The desserts are made of two round chocolate cakes with a layer of chocolate mousse between them. Each cake is 1\(\frac{3}{4}\) inches tall and the mousse is \(\frac{1}{2}\) inch thick. The diameter of each cake is 3 inches.

a. Sketch a diagram of Doris’ miniature chocolate dessert.
b. Doris only frosts the sides and top of each dessert after they are put together. Find the surface area of each dessert that needs to be frosted.
c. Doris has an order for 15 miniature chocolate desserts. How many square inches of dessert will need frosted to complete this order?
d. One batch of frosting covers 80 square inches. How many batches of frosting will she need to make to complete her order?

**REVIEW**

Find the surface area of each prism.

19.

20.

21.

22. A hexagon with an area of 165.6 square meters is the base of a 12 meter tall prism. Each side of the hexagon is 8 meters.

a. Find the lateral area of the hexagonal prism.
b. Find the total surface area of the prism.

23. What is the surface area of the prism formed by this net?
Create a set of net stencils a student could use to make their own solids. The set of nets should include:

- two different rectangular prisms
- a triangular prism
- a pentagonal or hexagonal prism
- a square pyramid
- a pentagonal or hexagonal pyramid
- two different cylinders

For a final presentation, include the stencils and a sample set of solids made from the stencils.

Suzy found a mix of rectangular fabric scraps in her mother's sewing box. One was 21 inches by 20 inches. Another was 20 inches by 10 inches. The last piece was 12 inches by 16 inches. She wants to wrap her gifts for her friends with the fabric pieces. The gift containers are shown below.

**Step 1:** Which piece of fabric should she use to wrap each gift? Explain your reasoning.

**Step 2:** Assume the fabric does not need to overlap at all when wrapping the gifts. Determine how much fabric would be left over after wrapping the three gifts with the fabric. Use 3.14 for π, when necessary.

**Step 3:** Suzy decided she needs 10% more fabric than the surface area to account for the overlap when wrapping the gifts. How much fabric will be left over when she uses 10% more fabric?
A pyramid is a solid with a polygonal base and triangular lateral faces that meet at a vertex. In this lesson, you will work with regular pyramids. The base of a regular pyramid is a polygon with sides of equal length and angles of equal measure. The slant height of a pyramid is the height of a lateral face. The variable \( l \) is used to represent slant height. The net of a square pyramid is shown below.

**EXPLORE!**

Tyrisha's family is planning a barbecue on the beach over Spring Break. They want to make a tent out of a tarp in case it rains. Tyrisha decides the tent should be an enclosed square pyramid.

**Step 1:** Draw a two-dimensional pattern (net) for Tyrisha's tent. The sides of the base will be 10 feet. The slant height of the tent will be 8 feet.

**Step 2:** Find the area of the base of the tent.

**Step 3:** Find the area of each lateral face of the tent.

**Step 4:** How much tarp is needed to exactly cover the lateral faces of the tent?

**Step 5:** Tyrisha's brother, Tyrone, says he would not need a net to find the lateral area. He says, “The lateral area of the pyramid is one-half of the perimeter of the base times the slant height.” Check his statement. Do you agree or disagree with Tyrone?

**Step 6:** Tyrone's formula only helps calculate the lateral area. They plan to enclose the entire space by putting tarp down for the base of the tent. Write a formula that will help you find the entire surface area of the tent.

**Step 7:** Tyrisha decides her tent should be a bit larger. The sides of the base of the larger tent will be 12 feet. The slant height of the tent will be 9 feet. Find the total amount of tarp needed. Explain your methods.
Lesson 14 ~ Surface Area Of Regular Pyramids

**Lateral Surface Area of a Regular Pyramid**

The lateral area (LA) of a pyramid is equal to half the perimeter (P) of the base times the slant height (l) of the pyramid.

\[ LA = \frac{1}{2}Pl \]

**Total Surface Area of a Regular Pyramid**

The surface area of a pyramid is equal to the sum of the lateral area (LA) and the area of the base.

\[ SA = LA + B \]

\[ SA = \frac{1}{2}Pl + B \]

---

**Example 1**

Find the surface area of the regular pentagonal pyramid given that the base area is 139 square centimeters.

**Solution**

Find the perimeter of the base.

\[ 9(5) = 45 \]

Locate the slant height of the prism.

\[ l = 12 \]

Use the surface area formula for a pyramid.

\[ SA = \frac{1}{2}Pl + B \]

Substitute all known values for the variables.

\[ SA= \frac{1}{2}(45)(12) + 139 \]

Multiply, then add.

\[ SA = 270 + 139 = 409 \]

The surface area of the pyramid is 409 square centimeters.

---

**Example 2**

Find the surface area of the square pyramid.

**Solution**

Find the perimeter of the base.

\[ 10.3(4) = 41.2 \]

Find the area of the base.

\[ 10.3(10.3) = 106.09 \]

Locate the slant height of the prism.

\[ l = 8 \]

Use the surface area formula for a pyramid.

\[ SA = \frac{1}{2}Pl + B \]

Substitute all known values for the variables.

\[ SA = \frac{1}{2}(41.2)(8) + 106.09 \]

Multiply, then add.

\[ SA = 164.8 + 106.09 = 270.89 \]

The surface area of the pyramid is 270.89 square inches.
EXERCISES

Determine the number of lateral faces on each pyramid.

1. octagonal pyramid
2. square pyramid
3. hexagonal pyramid
4. heptagonal pyramid
5. triangular pyramid
6. pentagonal pyramid

7. \( l = 10 \text{ cm} \)
   - a. Draw a net of the pyramid.
   - b. Find the area of each figure in the net.
   - c. Find the surface area of the pyramid.

8. \( l = 20 \text{ cm} \)
   - a. Draw a net of the pyramid.
   - b. Find the area of each figure in the net.
   - c. Find the surface area of the pyramid.

9. The base of a regular pentagonal pyramid has a perimeter of 60 feet. The slant height of the pyramid is 9 feet. Find the lateral area of the pyramid.

10. A square pyramid has a base edge that measures 8 meters and a slant height of 30 meters.
    - a. Find the perimeter of the base.
    - b. Find the lateral area.

Find the lateral area of each pyramid.

11. Slant height = 11

12. 

Find the surface area of each pyramid.

13. 

14. 

Lesson 14 – Surface Area Of Regular Pyramids
15. A regular triangular pyramid has a slant height of 10 inches. The perimeter of the base is 24 inches. The base of the pyramid has an area of 27.7 square inches.
   a. Find the lateral area of the pyramid.
   b. Find the surface area of the pyramid.

16. Terry made game pieces in the shape of square pyramids. Each piece has a base edge of 2 cm and a slant height of 4 cm. He will paint all of the pieces. He needs to know how much paint he needs.
   a. Find the surface area of one game piece.
   b. Each game has 24 game pieces. Find the total surface area of one set of game pieces.
   c. He wants to make 12 games. What is the total surface area for all 12 games?
   d. A can of paint covers 400 square centimeters. How many cans of paint will he need?

17. A regular hexagonal pyramid has a base area of 392.9 square feet. The sides of the hexagon are 12.3 feet long. The slant height of the pyramid is 15.9 feet. What is the surface area of the pyramid?

18. A square pyramid has a perimeter of 50 inches and a slant height of 15\(\frac{3}{4}\) inches.
   a. Find the lateral area of the pyramid.
   b. What is the length of one side of the base?
   c. Find the area of the base.
   d. Find the surface area of the pyramid.

19. A square pyramid has a base area of 64 cm\(^2\). The slant height of the pyramid is 7 cm.
   a. Find the length of one side of the base.
   b. Find the perimeter of the base.
   c. Find the lateral area of the pyramid.
   d. What is the surface area of the pyramid?

**REVIEW**

Calculate the surface area of each solid.

20. [Image of a cylinder]
21. [Image of a rectangular prism]
22. [Image of a cube]

23. How many edges does a hexagonal pyramid have?
24. How many vertices are on a pentagonal prism?
25. How many lateral faces are on an octagonal prism?
**Lesson 14 ~ Surface Area Of Regular Pyramids**

**Tic-Tac-Toe ~ A Solid Album**

Geometric solids are all around you in the world. Create an album of solids. Include a picture of a solid on each page of your album. The pictures can be photographs, newspaper or magazine clippings or pictures printed off the internet. Each page of the album should also include the following:

- A description of the object (what it is and where it can be found).
- The geometric name of the solid.
- The number of lateral faces, bases, edges and vertices, if applicable.
- Source of the picture (i.e. name of magazine, internet site, etc).

The album should include at least 10 pages with a front and back cover. At least one of each type of solid (prism, pyramid, cylinder, cone and sphere) should be included.

**Tic-Tac-Toe ~ Not Regular**

The pyramids in Lesson 14 have been regular pyramids.

1. Define regular pyramid.

2. Explain why a rectangular pyramid is not regular.

3. Draw a net of a rectangular pyramid. Cut out the net to make sure it forms a solid.

4. Measure needed lengths to the nearest tenth of a centimeter. Find the area of each figure in your net.

5. Find the surface area of your pyramid.

6. Explain why the formula for a regular pyramid \( SA = \frac{1}{2}Pl + B \) will not work for a rectangular pyramid.

7. Write a formula for calculating the surface area of a rectangular pyramid.

8. Clearly display all of your answers on a sheet of paper. Tape your net to your answer sheet.
A cone is a solid formed by one circular base and a curved surface which connects the base and the vertex. The formula for the lateral area of a cone is similar to the formula for the lateral area of a pyramid. The surface areas of pyramids and cones are found by adding the lateral area and the area of the base.

\[
\text{Lateral Area} = \frac{1}{2}(\text{perimeter of the base})(\text{slant height})
\]

\[
\text{Surface Area} = \frac{1}{2}(\text{perimeter of the base})(\text{slant height}) + \text{Area of the base}
\]

\[
\text{Lateral Area} = \frac{1}{2}(\text{circumference of the base})(\text{slant height})
\]

\[
\text{Surface Area} = \frac{1}{2}(\text{circumference of the base})(\text{slant height}) + \text{Area of the base}
\]

**EXAMPLE 1**

Jenny serves snow cones by putting flavored ice in a paper cone-shaped cup. Find the lateral area of the snow cone cup. Use 3.14 for \(\pi\).

**Solution**

Use the lateral area for a cone.

Substitute all known values for the variables.

Multiply.

The lateral area of the snow cone cup is about 251.2 square centimeters.
**EXAMPLE 2**  
Find the surface area of the cone. Use 3.14 for \( \pi \).

**Solution**

- Find the length of the radius. \( 12 \div 2 = 6 \)
- Find the area of the base.  
  \[ \pi r^2 \approx (3.14)(6)^2 \approx 113.04 \]
- Find the surface area for a cone.  
  \[ SA = \pi rl + B \]
- Substitute all known values for the variables.  
  \[ SA \approx (3.14)(6)(9) + 113.04 \]
- Multiply, then add.  
  \[ SA \approx 169.56 + 113.04 \approx 282.6 \]

The surface area of the cone is about 282.6 square inches.

**EXERCISES**

Find the lateral area of each cone. Use 3.14 for \( \pi \).

1. ![Diagram of cone with dimensions]
   
   2. ![Diagram of cone with dimensions]
   
   3. ![Diagram of cone with dimensions]

4. The circumference of a cone is 43.96 cm. The slant height of the cone is 6 cm.
   - a. Identify the measure that is missing in order to find lateral area.
   - b. Work backwards, using the circumference, to find the radius of the cone. Use 3.14 for \( \pi \).
   - c. Find the lateral area of the cone.

5. The radius of a cone is 10 cm. The slant height is 15 cm. Find the lateral area of the cone. Use 3.14 for \( \pi \).

6. Bobby works at an ice cream store. He needs to calculate the surface area of a sugar cone. Bobby knows the diameter of each cone is 2 inches. The slant height is 6 inches.
   - a. Find the lateral area of the cone.
   - b. Why would Bobby find the lateral area rather than the surface area of the ice cream cone?
Find the surface area of each cone. Use 3.14 for π.

7. \[ \text{Radius: 15 cm, Slant height: 2 cm} \]

8. \[ \text{Radius: 4 ft, Slant height: 6 ft} \]

9. \[ \text{Radius: 9.4 cm, Slant height: 21.7 cm} \]

The radius and slant height of each cone is given below. Complete each row of the table. Use 3.14 for π.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Slant height</th>
<th>Lateral area</th>
<th>Base area</th>
<th>Surface area</th>
</tr>
</thead>
<tbody>
<tr>
<td>10. 3 inches</td>
<td>5 inches</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11. 9 cm</td>
<td>2 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12. 7 ( \frac{1}{2} ) feet</td>
<td>8 ( \frac{3}{4} ) feet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13. 24.7 units</td>
<td>32.5 units</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Copy and complete the table by finding the surface area of each cone using different values of pi. If necessary, round to the nearest ten-thousandth.

<table>
<thead>
<tr>
<th>Radius</th>
<th>Slant height</th>
<th>Calculator ( \pi )</th>
<th>( \frac{22}{7} )</th>
<th>( \pi ), Exact answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>14. 7 yards</td>
<td>10 yards</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15. 28 cm</td>
<td>6 cm</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. Xing needs to wrap a necklace in a cone-shaped container. Find the least amount of wrapping paper needed if the container has a diameter of 8 cm and a slant height of 6 cm. Use 3.14 for π.

17. The lateral area of a cone is 675.1 square units. The radius of the cone is 20 units. Find the slant height. Use 3.14 for π.

18. The lateral area of a cone is 118.2. The slant height is 9.4.
   a. Use the formula for lateral area to find the radius. Use 3.14 for π.
   b. Find the area of the base of the cone. Use 3.14 for π.

**REVIEW**

Determine what value of π is most appropriate for each situation. Explain your choice.

19. Gabe wrote a program for a machine to make precise cuts.

20. Marcus found a rough estimate of the space the base of a circular swimming pool will cover in his backyard.
21. Raquelle made a circular placemat. She purchased some ribbon for the edge. The radius of the placemat is 7 inches.

22. Shelley found the circumference of a circle with a radius of 4.6 centimeters.

23. Tham bought mulch for his circular flower bed. He estimated the diameter of the flower bed to be 15 feet.

---

**Tic-Tac-Toe ~ All the Same**

Step 1: Find the surface area of the solid.

Step 2: Draw four more solids that each have approximately the same surface area as the prism in Step 1. Label all necessary dimensions. Use 3.14 for π, when necessary. Include the following:

- another prism
- a cylinder
- a pyramid
- a cone

Step 3: Write 1-2 paragraphs explaining the processes you used to determine the dimensions of your solids. Which solid was the most difficult to create? Which was the easiest? Why?

---

**Tic-Tac-Toe ~ Research Says...**

There are many landmarks and buildings that are solids. For example, the Pentagon in Washington, DC is a pentagonal prism with the center removed. The Pyramid du Louvre is a glass and metal pyramid that is an entrance to a museum in Paris, France. Research a famous landmark or building that is a solid. Write a one page paper about the importance of the landmark. On a separate sheet of paper, calculate the surface area of the landmark.
A composite solid is a three-dimensional figure made of more than one solid. To find the surface area of a composite solid you must determine which two-dimensional figures are on the surface of the solid. Some surfaces are covered up when a solid is connected to another solid. Each picture below represents a composite solid.

**Example 1**

Find the surface area of the composite solid.

**Solution**

This composite solid is made of a square prism with a square pyramid on top.

Identify the parts of the solid on the surface.

- 1 base
- LA of prism
- LA of pyramid

Find the area of the base. \((10)(10) = 100\)

Find the lateral area of the prism. \(LA = Ph = 40(32) = 1280\)

Find the lateral area of the pyramid. \(LA = \frac{1}{2} Pl = \frac{1}{2} 40(12) = 240\)

Find the sum of all three parts. \(SA = 100 + 1280 + 240 = 1620\)

The surface area of the composite solid is 1620 square feet.

**Surface Area of Composite Solids**

1. Identify the different types of figures that make up the solid.
2. Identify which parts of each figure are on the surface of the solid.
3. Calculate the areas of all parts on the surface.
4. Find the sum of the areas.
Lesson 16 ~ Surface Area Of Composite Solids

**EXAMPLE 2**  
Find the surface area of the composite solid. Use 3.14 for \( \pi \).

This composite solid is made of a cylinder and two cones.

Identify the parts of the solid on the surface.  
LA of cylinder  
LA of cone on right  
LA of cone on left

Find the lateral area of the cylinder.  
\[ LA = 2\pi rh \]  
\[ \approx (2)(3.14)(2)(3) \approx 37.68 \]

Find the lateral area of the cone on the right.  
\[ LA = \pi rl \]  
\[ \approx (3.14)(2)(4) \approx 25.12 \]

Find the lateral area of the cone on the left.  
\[ LA = \pi rl \]  
\[ \approx (3.14)(2)(3) \approx 18.84 \]

Find the sum of all three parts.  
\[ SA \approx 37.68 + 25.12 + 18.84 \approx 81.64 \]

The surface area of the composite solid is about 81.64 square meters.

**EXERCISES**

Find the surface area of each composite solid. When necessary, use 3.14 for \( \pi \).

1.  
2.  
3.  
4.  
5.  
6.
7. Sarah celebrated her fifth birthday. She ate at her favorite restaurant. She ordered a soda pop. The soda pop came in a cup shaped like a cylinder with a cone top. The cylinder part of the cup was 6 inches tall and the slant height of the top was 2 inches. The radius of the cup was 2 inches. What was the surface area of the cup?

8. James wants to paint his grain silo. The diameter of the silo is 8 meters. The height of the cylindrical part is 12 meters. The slant height of the cone top is 4.5 meters.
   a. Calculate the surface area of the grain silo. Use 3.14 for \( \pi \).
   b. A five-gallon bucket of paint covers 20 square meters. How many buckets of paint will James need?

9. A greenhouse is generally a wood or metal frame covered in clear plastic. Calculate how much plastic will be needed for the greenhouse shown.

10. Some pencils are in the shape of a hexagon. The hexagonal pencils are 4.5 \( mm \) on each side and 20 \( cm \) long.
    a. Convert 20 centimeters to millimeters.
    b. Find the lateral area of an unsharpened pencil.
    c. When the pencil is sharpened, what parts of solids make up the new lateral area?

11. Jennifer designed her perfect wedding cake. She wants to have 3 layers with smooth white frosting on the cake. The first layer will have a 24-inch diameter, the second layer will have an 18-inch diameter and the top layer will have a 10-inch diameter. Each layer will be 6-inches tall. How many square inches of frosting will show on the surface of the cake?

**REVIEW**

12. Name each solid below.

   a. 
   b. 
   c. 
   d. 

13. Identify the number of edges and vertices in each solid in Exercise 12.
Name each of the following using $\odot P$.

14. two chords
15. three radii
16. longest chord
17. a central angle less than 90°
18. a central angle more than 90°

There are five basic solids: prisms, cylinders, pyramids, cones and spheres. The surface area of a sphere is not the sum of the lateral area and the base area. The formula is given below.

**Surface Area of a Sphere**
The surface area of a sphere is the product of 4, $\pi$ and the radius squared.

$$SA = 4\pi r^2$$

1. Explain why the formula $SA = LA + B$ does not work for a sphere.

2. Use resources to find the radius of the eight planets in our solar system.

3. Assume each planet is a perfect sphere. Calculate the surface area of each planet in square miles. Use 3.14 for $\pi$.

4. Create a booklet showing the eight planets and their surface areas. Include at least one other interesting fact about each planet.
Lesson 10 ~ Three-Dimensional Figures

Identify the number of faces, lateral faces, bases, edges and vertices in each solid.

1. 

2. 

3. 

Name the solid that fits each description.

4. a can of hair spray
5. a basketball
6. a prism with 8 sides
7. a pyramid with 3 lateral faces
8. a solid with 8 vertices
9. a solid with two bases
Lesson 11 ~ Drawing Solids

10. What is a net?

Draw a net for each solid.

11. [Diagram of a cube]

12. [Diagram of a tetrahedron]

13. [Diagram of a cylinder]

Draw a diagram of each solid.

14. cone

15. trapezoidal prism

16. pentagonal pyramid

Lesson 12 ~ Surface Area of Prisms

Find the surface area of each prism.

17. The base of a cereal box is 14 square inches. The lateral area of the box is 162 square inches.

18. The lateral area of a hexagonal prism is 60 $m^2$. The base is 10 $m^2$.

19. [Diagram of a rectangular prism with dimensions 5 cm x 12 cm x 19 cm]

20. [Diagram of a rectangular prism with dimensions 3 ft x 10 ft x 14 ft]

21. [Diagram of a triangular prism with dimensions 3 ft x 3 ft x 5 ft and 5 cm x 5 cm x 8 cm]

22. Kendra’s toy chest needs a paint job. The toy chest is 3 feet long, 1.5 feet wide and 2 feet tall. Find the surface area of the toy chest.

23. The surface area of a triangular prism is 35 $in^2$. If the lateral area is 30 $in^2$, find the area of one base.

24. Jahzara took the Oregon State Assessment. In one problem, the dimensions of a box were 6 m by 10 m by 8 m. She used the formula $SA = 2(lw + wh + lh)$. Use this formula to calculate the surface area of the box.
Lesson 13 ~ Surface Area of Cylinders

Find the surface area of each cylinder. Use 3.14 for $\pi$.

25. 

26.  

27. The lateral area of a cylinder is 75 square meters. The radius of the base is 10 m. Find the surface area of the cylinder.

28. A can of beans has a radius of 2 inches and a height of 5 inches. Find the area of the paper label.

29. How many square inches of aluminum are needed to cover the surface of the can of beans in Exercise 28?

Lesson 14 ~ Surface Area of Regular Pyramids

30. How is the slant height of a pyramid different than the height of a pyramid?

31. Each side of a pentagonal pyramid is 7 centimeters. Find the lateral area of the pyramid if the slant height is 10 centimeters.

Find the surface area of each pyramid.

32. 

33. 

34. The Great Pyramid in Egypt was originally 754 feet on each side and the slant height was approximately 610 feet. Calculate the original lateral area of the Great Pyramid.

35. A square pyramid has a perimeter of 100 inches and a slant height of 10 inches.
   a. Find the lateral area of the pyramid.
   b. What is the length of one side of the base?
   c. Find the area of the base.
   d. Find the surface area of the pyramid.
Lesson 15 ~ Surface Area of Cones

36. Identify an occasion when it may be necessary to only calculate the lateral area of a cone.

Find the lateral area of each cone. Use 3.14 for π.

37. Radius = 1 cm
   Slant height = 4 cm
38. \( r = 12.5 \text{ ft} \)
   \( l = 8 \text{ ft} \)
39. Diameter = 22.4 m
   \( l = 35.7 \text{ m} \)

Find the surface area of each cone. Use 3.14 for π.

40.
41. Diameter = 15 m
42. Diameter = 1.5 in

43. A plastic company coats a specialized tool. The tool is cone-shaped. It has a diameter of 22 mm and a slant height of 35 mm. Find the amount of plastic coating needed to cover the lateral area of the cone. Use 3.14 for π.

Lesson 16 ~ Surface Area of Composite Solids

44. What is a composite solid?

45. Sketch a composite solid. Identify the parts of the solid that are on the surface.

Find the surface area of each composite solid. When necessary, use 3.14 for π.

46.
47.

48. An ornament is made of two cones glued together at their bases. The radius of each cone is 1.5 inches. The slant height of each cone is 3 inches. Find the surface area of the ornament.

49. Mark built a greenhouse in his backyard. He copied his neighbor’s greenhouse shown here. How much plastic did it take to cover the greenhouse?
RICK
MASON
McMinnville, Oregon

I am a mason. Masonry is one of the many trades that contribute to the completion of a construction project. Masonry projects include concrete block buildings and walls, brick-laying, stone working, and fireplaces. I specialize in custom brick and stone work. My company contracts work with both general contractors and individual homeowners.

A masonry project requires the use of math from start to finish. I use basic math and Algebra skills so often that they become second nature. Calculating measurements, spacing, and areas are a constant throughout my work day. Without accurate measurements, my finished work will not look good. Even before a project can begin, I use math skills to estimate the potential cost of doing a certain project.

Many masons have learned the trade by working with an experienced crew. Working with a crew gives you good on the job training and valuable experience. This is how I became a mason. Other masons attend community colleges or trade schools to learn how to do the work. Some unions even offer training and apprentice programs for people wanting to become masons.

Masons can expect to earn between $18,000 to $45,000 per year depending on expertise, experience and location. A mason may decide to work for a commercial contractor who builds large buildings, or they might work for a residential contractor and work on individual homes.

I enjoy my job because I am able to design and build a project that will be used and appreciated for generations. I welcome the challenge of working in a variety of settings from restoring historic buildings to stonework on vineyard estates. I also enjoy working in different locations like California, Montana or even Alaska. What I enjoy most about my job is seeing the results of my work at the end of each day.