BLOCK 3 ~ LINEAR EQUATIONS

USING LINEAR EQUATIONS

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WORD WALL

EQUIVALENT

DISCRETE

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SLOPE-INTERCEPT FORM

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In Lesson 12 you were introduced to linear functions. **Slope-intercept form** is the most common equation used to represent a linear function. It is called this because the slope and the $y$-intercept are easily identified.

**Slope - Intercept Form of a Linear Equation**

$$y = mx + b$$

The slope of the line is represented by $m$.
The $y$-intercept is $b$.

- Slope ($m$) is also called the rate of change. The slope gives you the rise over the run of the line.
- The $y$-intercept is also called the start value. The $y$-intercept is the location the line crosses the $y$-axis. The ordered pair for the $y$-intercept will be $(0, b)$.
- Equations in slope-intercept form may also be written $y = b + mx$.

Graphing an equation is a very important skill in mathematics because it is a visual representation of a mathematical equation. In this lesson you will learn how to graph an equation when it is presented in slope-intercept form.

**EXAMPLE 1**

Graph $y = \frac{1}{3}x - 2$. Clearly mark at least three points on the line.

**SOLUTION**

First determine the slope and $y$-intercept. $y = \frac{1}{3}x - 2$

$m = \frac{1}{3}$ and $b = -2$

Start by graphing the $y$-intercept on the coordinate plane. Use the slope to find at least two more points. Remember rise over run. Draw a straight line through the points. Put an arrow on each end.
Graph each linear equation. Clearly mark at least three points on each line.

\[ \text{a. } y = 6 - 2x \quad \text{b. } y = -\frac{7}{2}x + 1 \]

\[ \text{a. } y = 6 - 2x \rightarrow m = -2 \text{ and } b = 6 \]

\[ \text{b. } y = -\frac{7}{2}x + 1 \rightarrow m = -\frac{7}{2} = -\frac{7}{2} \text{ and } b = 1 \]

As you learned in Block 2, there are lines that have a slope of zero and other lines that have an undefined slope. The equations of these lines are unique.

**Zero slope**
\[ y = \text{constant} \]
\[ y = 2 \]

**Undefined slope**
\[ x = \text{constant} \]
\[ x = -1 \]
When a graph can be drawn from beginning to end without lifting your pencil, it is **continuous**. Some situations are modeled by linear equations but are not continuous. This could mean that it would not make sense to connect the points of the equation with a line.

For example, Jenna started with $50 in her savings account. She adds $10 each month. The linear equation that represents the balance in her savings account is \( y = 50 + 10x \). Since she only puts money in her account once a month, a line should not be drawn through the points on the graph. Only the whole numbers and 0 can be used as \( x \)-values. This graph is called **discrete** because it is represented by a unique set of points rather than a continuous line.

A graph can be continuous but limited to a certain quadrant or section of the graph. For example, Lamar types 40 words per minute. It would not make sense to graph points out of the first quadrant because he cannot type for a negative number of minutes or type a negative number of words. This graph is continuous because it can be drawn without lifting your pencil.

**EXERCISES**

Draw a coordinate plane for each problem. Graph the given equation. Clearly mark three points on the line.

1. \( y = \frac{1}{3}x - 3 \)  
2. \( y = 1 - 3x \)  
3. \( y = -\frac{2}{3}x + 6 \)

4. \( y = x + 2 \)  
5. \( y = 4 \)  
6. \( y = -5 + \frac{4}{3}x \)

7. \( x = -2 \)  
8. \( y = 5x \)  
9. \( y = -\frac{3}{2}x + 4 \)

10. Taylor's graphs of two different linear equations are seen below. His teacher told him both graphs were incorrect. Explain to Taylor, in complete sentences, why each of his graphs is not correct.

a. \( y = \frac{4}{3}x + 1 \)  

b. \( x = 3 \)
11. Daryl was given the linear equation $y = 1.5x + 2$. He was not sure how to graph this equation because its slope was a decimal. Follow the process Daryl decided to use.
   a. Copy the table and use the equation to fill in the output values.
   b. Graph your ordered pairs $(x, y)$ from the table on a coordinate plane. Draw a line through the points.
   c. Daryl was quite happy with the graph of the linear equation and was sure he had found the easiest way to deal with a linear equation which has a decimal slope value. Do you agree with him? Why or why not?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
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<tbody>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
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<tr>
<td>4</td>
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12. Graph the three linear equations on the same coordinate plane. In complete sentences, describe the similarities and differences of the three lines.
   
   $y = \frac{3}{4}x$
   
   $y = \frac{3}{4}x - 5$
   
   $y = \frac{3}{4}x + 2$

13. Create a linear equation that satisfies each condition. Graph your equations on a coordinate plane.
   a. Slope = $\frac{1}{3}$ and a negative $y$-intercept
   b. Slope = 0 and a $y$-intercept of 4
   c. A positive slope and a positive $y$-intercept
   d. A negative slope and a $y$-intercept of 0.

14. Mrs. Samuels warned her class that the linear equations shown below were the most-often missed problems on the linear equations test she gave to her class last year. Explain why you think each problem might have been missed and then graph each equation.
   a. $y = x - 3$
   b. $y = 4$
   c. $x = -4$
   d. $y = 6 - x$

15. Falls City, Oregon experienced a massive rainstorm from December 26th to December 30th in 1936. On the first day of the storm it rained 5.5 inches. It continued to rain 2.5 inches each day for the next four days.
   a. Fill in the table with the TOTAL rain that had fallen during the storm as each day passed.
   b. In this situation, which number represents the slope (or rate of change)?
   c. Determine the $y$-intercept.
   d. Write a linear equation that represents the total rainfall in Falls City based on the number of days the storm has lasted.
   e. If the storm had continued at the same rate for 10 days, what would have been the total rainfall?

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
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<tbody>
<tr>
<td>1</td>
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<td>3</td>
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<td>5</td>
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16. Dave was able to do 3 pull-ups before attending PE class. Each week of PE, the number of pull-ups he was able to do increased by 2.
   a. Write a linear equation representing the number of pull-ups, $y$, Dave was able to do in a given week, $x$.
   b. Would this graph be a continuous line? Why or why not?
   c. Graph the equation in the way that best models the situation.
Choose which of the following graphs is the best model for each situation. Explain your reasoning.

A. ![Graph A]

B. ![Graph B]

C. ![Graph C]

17. Ima started collecting coins. She adds 5 coins to her collection each week.

18. Todd runs at a rate of 5 miles per hour.

19. A continuous relationship where \( y \) is 5 times \( x \).

**REVIEW**

Find the slope of the line that passes through the given points.

20. (2, 9) and (6, 11)  
21. (1, 2) and (1, 6)  
22. (3, −1) and (5, 0)

23. (−2, 6) and (1, 9)  
24. (4, 5) and (0, 5)  
25. (1, 2) and (4, −3)

**Tic-Tac-Toe ~ x- and y-Intercepts**

When linear equations are written in standard form \((Ax + By = C)\), they can be graphed by finding the \(x\)- and \(y\)-intercepts and then drawing a line through those two points.

- In order to find the \(x\)-intercept, you must substitute 0 for \(y\) in the equation and then solve for \(x\).
- In order to find the \(y\)-intercept, you must substitute 0 for \(x\) in the equation and then solve for \(y\).

*For example:* Graph \(2x - 3y = 6\) using the intercept method:

<table>
<thead>
<tr>
<th>(x)-intercept</th>
<th>(y)-intercept</th>
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</thead>
<tbody>
<tr>
<td>(2x - 3(0) = 6)</td>
<td>(2(0) - 3y = 6)</td>
</tr>
<tr>
<td>(2x = 6)</td>
<td>(-3y = 6)</td>
</tr>
<tr>
<td>(x = 3)</td>
<td>(y = -2)</td>
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</table>

Graph each of the following using the intercept method.

1. \(5x + 4y = 20\)  
2. \(-3x + y = 6\)  
3. \(2x - 3y = 9\)  
4. \(-3x + 5y = 15\)  
5. \(4x - y = -8\)  
6. \(4x + 7y = 14\)  
7. Convert each of the equations in #1-6 into slope-intercept form using the method shown in Lesson 19. Use the slope-intercept equation to verify that each graph has the correct \(y\)-intercept and slope.
Stacey and Mario like to go to the coffee shop before school. They decided to conduct an experiment to study the rate at which their coffees cool when left untouched on the table. The graph below shows the information they gathered.

**Step 1:** What is the real world meaning of the point \( (0, 160) \)? How about the point \( (10, 120) \)?

**Step 2:** Use the slope formula, \( \frac{y_2 - y_1}{x_2 - x_1} \), to find the slope of the line. Does it matter which points from the graph you use in the formula?

**Step 3:** What is the real-world meaning of the slope?

**Step 4:** What is the \( y \)-intercept of this graph?

**Step 5:** Write an equation in slope-intercept form that represents this graph.

**Step 6:** Use your equation to determine the temperature of the coffee after 12 minutes.

**Step 7:** According to Stacey and Mario’s experiment, the coffee continued to cool at the same rate every minute that passed. Do you think the coffee will continue to cool at this rate if the coffee is left on the table for one hour? Verify your theory using your equation.

A linear equation can be written for a specific line if you know the slope and \( y \)-intercept. The \( y \)-intercept can be determined by locating the point where the graph crosses the \( y \)-axis \((0, b)\). The slope must be calculated using a slope triangle or the slope formula. Remember that when dealing with real-world graphs, the \( y \)-intercept is referred to as the start value and the slope is called the rate of change.
**EXAMPLE 1**

Determine the slope and $y$-intercept of each graph. Write the equation for each graph in slope-intercept form.

a. The line crosses the $y$-axis at 1 so $b = 1$.
The slope triangle shows that $m = \frac{2}{3}$.
The equation in slope-intercept form is: $y = \frac{2}{3}x + 1$.

b. The line crosses the $y$-axis at 4 so $b = 4$.
The slope triangle shows that $m = -\frac{1}{2}$.
The equation in slope-intercept form is: $y = -\frac{1}{2}x + 4$.

c. The line crosses the $y$-axis at $-4$ so $b = -4$.
The slope triangle shows that $m = \frac{0}{3} = 0$.
The equation in slope-intercept form is $y = 0x - 4$ which can also be written as $y = -4$.

Solutions

It is very useful to have equations for graphs that represent real-world situations because you can use the equation to predict future or past data.

**EXAMPLE 2**

Zach enjoys running each day after school. The graph below represents the distance Zach has traveled based on the number of minutes he has been running.

a. Find the slope-intercept equation that represents the situation shown on the graph.
b. Use your equation to determine how far Zach will have gone in 28 minutes.
c. Use your equation to determine how long it will take Zach to run 10 miles.

Solutions

a. The line crosses the $y$-axis at 0 so $b = 0$.
The two marked points are (8, 1) and (16, 2). Use the slope formula to calculate the slope.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{16 - 8} = \frac{1}{8}$$

The slope-intercept equation is $y = \frac{1}{8}x + 0$ or $y = \frac{1}{8}x$. 
b. Since the $x$-values represent minutes, substitute 28 for $x$ to determine how far Zach runs in 28 minutes:

\[
y = \frac{1}{8}x \\
y = \frac{1}{8}(28) \\
y = \frac{28}{8} = 3\frac{4}{8} = 3\frac{1}{2} = 3.5
\]

Zach runs 3.5 miles in 28 minutes.

c. Since the $y$-values represent miles, substitute 10 for $y$ to determine how long it will take Zach to run 10 miles.

\[
\begin{align*}
10 &= \frac{1}{8}x \\
\frac{8}{1} \cdot 10 &= \frac{1}{8}x \cdot \frac{8}{1} \\
80 &= x
\end{align*}
\]

Zach’s 10 mile run will take 80 minutes (one hour and twenty minutes).

Stacey and Mario found it was easy to write a slope-intercept equation from the graph because all they had to do was find the $y$-intercept and slope and put it into the form $y = mx + b$. They found the equation for their coffee experiment in the Explore! to be $y = -4x + 160$. They tested their formula by substituting values for the temperature of the coffee to see if it matched the number of minutes shown on the graph.

Temperature = 150°F $\rightarrow$ Since the $y$-values represent the temperature, substitute 150 for $y$.

\[
\begin{align*}
150 &= -4x + 160 \\
-160 &= -4x \\
-10 &= 4x \\
2.5 &= x
\end{align*}
\]

The coffee was 150° after 2.5 minutes.

Stacey and Mario used their formula to predict when their coffee would freeze.
Temperature = 32°F (freezing) $\rightarrow$ Since the $y$-values represent temperature, substitute 32 for $y$.

\[
\begin{align*}
32 &= -4x + 160 \\
-160 &= -4x \\
-128 &= 4x \\
32 &= x
\end{align*}
\]

According to the equation, the coffee would freeze in 32 minutes.

Stacey and Mario decided their formula only works for the first 10 minutes or so. It is not likely that coffee is going to reach a freezing temperature while it is sitting on the table.
Identify the slope and \( y \)-intercept of each graph. Write the corresponding linear equation in slope-intercept form.

1.  
2.  
3.  

4.  
5.  
6.  

7. Skyler’s total savings are shown on the graph.  
   a. Find the slope-intercept equation that represents the graph.  
   b. Use your equation to determine how much Skyler will have in his savings account after 18 months.  
   c. Use your equation to determine how many months it will take before Skyler has $212 in his savings.

8. Javier owns a car rental company. He provides the graph seen at left for his customers to see the price for renting a sedan based on the number of miles they drive.  
   a. Find the equation (in slope-intercept form) that represents the amount Javier charges based on the number of miles driven.  
   b. Determine the amount a customer will have to pay if she rents a sedan and drives it 120 miles.  
   c. Leticia rented a sedan from Javier. When she returned it, her bill was $38.50. How many miles did she drive?
9. At two different times during the summer, Kirsten measured the height of a sunflower she had planted in May. She measured it when she first planted the flower and then again 3 weeks later.

   a. Find the slope-intercept equation that represents the height of Kirsten’s flower based on the number of weeks since she planted it.
   b. Use your equation to determine exactly how tall the sunflower will be after 8 weeks.
   c. Use your equation to determine how many weeks have passed if the plant is 47 inches tall.

10. Luke drained his 20-gallon fish tank. At two different times, he measured the amount of water left in the tank. He graphed the information on the graph shown at right.

   a. Find the slope-intercept equation that represents the number of gallons left in the fish tank since he began draining it.
   b. Use your equation to determine how much water will be left in the tank after 10 minutes.
   c. When will the water be completely drained from the tank?

11. Use the graph at left to answer the following questions.

   a. Find the slope-intercept equations for lines $m$, $n$ and $p$.
   b. What do the three equations have in common?
   c. What geometry term can be used to describe the relationship between these three lines?
**Lesson 17 ~ Writing Linear Equations For Graphs**

**Review**

12. Evaluate the following expressions when $x = 3$ and $y = -4$
   a. $2x + 5y$
   b. $-4x - 6y$
   c. $\frac{1}{2}y - 5x$

State whether each equation is true or false for the values of the variables given.

13. $y = 3x + 1$ when $x = 2$ and $y = 6$
14. $y = \frac{4}{3}x - 4$ when $x = 6$ and $y = 4$
15. $y = -2x + 7$ when $x = 5$ and $y = -3$
16. $y = \frac{1}{4}x$ when $x = 10$ and $y = 2$
17. $y = 4 - x$ when $x = 3$ and $y = 1$
18. $y = 2 - \frac{1}{2}x$ when $x = 1$ and $y = 2$

---

**Tic-Tac-Toe ~ Graphing Design**

Lines are used in many types of artwork. Use a large sheet of graph paper to create a piece of artwork.

**Step 1:** Draw a coordinate plane that includes all four quadrants.

**Step 2:** Create a design using at least 15 different lines. Make sure over two-thirds of the lines are not vertical or horizontal.

**Step 3:** Write the equations for each line on the back of your piece of artwork.

**Step 4:** Color your artwork and sign the bottom right corner.

---

**Tic-Tac-Toe ~ Parallel or Perpendicular Lines**

Perpendicular lines are lines that intersect at a 90° angle. Parallel lines never intersect. Each pair of lines given below is either parallel or perpendicular.

**SET #1**
- $y = 2x + 3$
- $y = 2x - 4$

**SET #2**
- $-3x + 2y = 6$
- $y = -\frac{2}{3}x - 4$

**SET #3**
- $y - x = 5$
- $y = x - 2$

**SET #4**
- $-x + 2y = -4$
- $4x + 2y = 8$

**SET #5**
- $y = \frac{1}{3}x + 3$
- $y = -3x - 1$

**Step 1:** If necessary, convert each equation into slope-intercept form using the method shown in Lesson 19.

**Step 2:** Graph each pair of equations on the same coordinate plane.

**Step 3:** State whether each pair of lines is parallel or perpendicular.

**Step 4:** After completing all five sets of graphs, develop a hypothesis on how to use the slope-intercept equation to determine if lines are parallel or perpendicular without graphing. Explain how you arrived at your hypothesis.
There are two pieces of information you need to be able to write an equation in slope-intercept form: the slope and the $y$-intercept. You learned how to determine the equation when given a graph of the linear equation. In this lesson you will find equations for specific lines when given different pieces of information about the lines.

When you are given the slope and the $y$-intercept for a line you need to insert the information into $y = mx + b$ for the appropriate variables.

**EXAMPLE 1**

Write the equation of a line that has a slope of $-2$ and $y$-intercept of 5.

**Solution**

Write the general slope-intercept equation. $y = mx + b$

Substitute $-2$ for $m$ since $m$ represents the slope. $y = -2x + b$

Substitute 5 for $b$ since $b$ represents the $y$-intercept. $y = -2x + 5$

The equation is $y = -2x + 5$.

When you are not directly given the slope and $y$-intercept, there are steps you can follow to find both the slope and $y$-intercept. Once you have both the slope and $y$-intercept, you can write a linear equation in slope-intercept form.

**WRITING A LINEAR EQUATION WHEN GIVEN KEY INFORMATION**

1. Find the slope ($m$) of the line.
2. Find the $y$-intercept ($b$) of the line. If necessary, substitute the slope for $m$ and one ordered pair $(x, y)$ for the corresponding variables in the equation $y = mx + b$. Solve for $b$.
3. Write the equation in the form $y = mx + b$. 

Lesson 18 ~ Writing Linear Equations From Key Information
**EXAMPLE 2**

Write the equation of a line that has a slope of \( \frac{4}{3} \) and goes through the point \((-3, 1)\).

**Solution**

The slope is given.

Write the slope-intercept equation with the slope.

Find the \(y\)-intercept, \(b\), by substituting the given point \((-3, 1)\) for \(x\) and \(y\) in the slope-intercept equation.

Solve for \(b\).

Write the equation by substituting \(m\) and \(b\).

Check by graphing.

If you are given two points on a line, first find the slope using the slope formula or a slope triangle. Then follow the process in Example 2 to write the slope-intercept equation.

**EXPLORE!**

A triangle consists of three line segments. A segment is a portion of a line.

**Step 1:** Find the equation of the line that contains \(\overline{AB}\).

**Step 2:** Find the equation of the line that contains \(\overline{AC}\).

**Step 3:** Find the equation of the line that contains \(\overline{BC}\).

**Step 4:** Ryan made his own triangle. He chose three points and wants you to find the equations of the three lines that make up his triangle. His points are \((3, 3), (-2, -7)\) and \((-7, -2)\). Can you find the three linear equations that intersect to make his triangle?

**Step 5:** The points where the line segments meet in a triangle are called the vertices. Graph your three lines on a piece of graph paper using the slope and \(y\)-intercepts from your equations. Does the triangle that is formed have the same three vertices that Ryan chose?
EXERCISES

Write an equation in slope-intercept form when given the slope and y-intercept.

1. slope = $\frac{6}{5}$, y-intercept = 8
2. slope = $-4$, y-intercept = 1
3. slope = 1, y-intercept = 2
4. slope = 0, y-intercept = −3

5. In 2000, the population of Oregon was approximately 3,400,000 people. During the next six years, the population increased by approximately 50,000 people each year.
   a. Write an equation in slope-intercept form that represents the population, $y$, of Oregon in terms of the number of years, $x$, since 2000.
   b. Estimate the population of Oregon in 2020 if this trend continues.

Write an equation in slope-intercept form when given the slope and one point on the line.

6. slope = 2, goes through the point (1, −4)
7. slope = $-\frac{3}{4}$, goes through the point (4, 2)
8. slope = −1, goes through the point (−2, 3)
9. slope = $\frac{5}{2}$, goes through the point (−6, −10)
10. slope = $\frac{1}{2}$, goes through the point (3, 4)
11. slope = 0, goes through the point (11, 8)

12. One Portland taxi company charges an initial fee plus $0.10 for each minute of the ride. Tammy was in the taxi for 14 minutes. The cost was $5.40. Let $x$ represent the number of minutes and $y$ represent the total cost of the taxi ride.
   a. Identify the slope and one ordered pair from the information given.
   b. Find the equation of the line that fits this information.
   c. Joe uses this taxi company for a 30 minute ride. How much should he expect to pay?

Write an equation in slope-intercept form when given two points.

13. goes through the points (1, 2) and (3, 8)
14. goes through the points (−5, 9) and (4, 0)
15. goes through the points (6, 9) and (−3, 6)
16. goes through the points (8, −4) and (−2, 1)
17. goes through the points (10, 6) and (0, −2)
18. goes through the points (4, −5) and (1, −2)
19. goes through the points (7, 3) and (0, 3)
20. goes through the points (1, −5) and (1, 4)
21. At 2 weeks old, Bob's baby sister weighed 9 pounds. When she was 8 weeks old, she weighed 12 pounds. Let \( x \) represent how old the baby is in weeks and \( y \) represent the baby’s weight in pounds.
   a. Write two ordered pairs that use the data about Bob’s sister.
   b. Find the equation of the line that goes through these two points.
   c. If Bob’s sister continues to grow at this rate, how much will she weigh when she is 20 weeks old? Is this reasonable? Why or why not?

22. Four line segments make the four sides of a quadrilateral on the coordinate plane to the left. Find the equations of the lines containing each side: \( \overline{AB} \), \( \overline{BC} \), \( \overline{CD} \), \( \overline{AD} \). Are there any similarities in the equations for lines \( \overline{AB} \) and \( \overline{CD} \)? How about \( \overline{BC} \) and \( \overline{AD} \)?

23. A raft rental company on the Deschutes River rents rafts for a set fee plus an additional charge per hour. Francis asked two different people how many hours they had rented their rafts for and how much it cost. One rented a raft for 6 hours and paid $32. Another rented a raft for 11 hours and paid $47. Let \( x \) represent the length of time in hours and let \( y \) represent the total cost.
   a. Write two ordered pairs that use the data Francis collected.
   b. Find the equation of the line that goes through these two points.
   c. What number in the linear equation represents the amount of the set fee?
   d. What is the real world meaning of the slope in this equation?
   e. How much will someone pay for a raft rental from this company if he only keeps the raft for 4 hours?

**REVIEW**

Write the linear equation for each graph in slope-intercept form.

24.

25.

26.
Draw a coordinate plane for each problem. Graph each equation. Clearly mark three points on the line.

27. \( y = \frac{5}{3}x - 4 \)
28. \( y = -4x + 5 \)
29. \( x = 3 \)

Write the slope-intercept equation that matches the data.

30. Hours | Distance Traveled
0 | 45
1 | 75
2 | 105
3 | 135
4 | 165

31. Days | Plant Height
1 | 3.3
2 | 3.6
6 | 4.8
8 | 5.4
10 | 6.0

32. | Cups of Coffee | Profit
|---|---|---|
0 | 0 | -15
4 | 4 | -7
9 | 9 | 3
15 | 15 | 15
100 | 100 | 185

Tic-Tac-Toe ~ Lines of Best Fit

You have learned how to take real-world data and graph the data points on a scatter plot. When real-world data is collected there is usually not a single line that passes through all of the data points, but you can often see a linear pattern on the scatter plot. In this activity, you will find a line that best fits the data you are given. This line is called the line of best fit. The steps to finding a line of best fit are shown in the box below.

Finding a Line of Best Fit
1. Plot the data points and determine the general direction of the data.
2. Draw a line in that general direction that has approximately the same number of points above and below the line.
3. Locate two points on your line. Approximate the x- and y-coordinates for each point (these do not need to be data points from the original data).
4. Find the slope-intercept equation of the line.

The data in the table shows the fare costs for different lengths of rides on the city bus.

1. Graph the data points.
2. Draw a line of best fit.
3. Find the equation for the line of best fit.
4. Use your equation to predict the cost to ride this city bus 50 miles.
5. Use your equation to predict how far you could ride the bus for $10.

<table>
<thead>
<tr>
<th>Miles</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$1.00</td>
</tr>
<tr>
<td>5</td>
<td>$1.75</td>
</tr>
<tr>
<td>10</td>
<td>$2.00</td>
</tr>
<tr>
<td>12</td>
<td>$2.50</td>
</tr>
<tr>
<td>16</td>
<td>$2.50</td>
</tr>
<tr>
<td>20</td>
<td>$3.25</td>
</tr>
<tr>
<td>26</td>
<td>$3.50</td>
</tr>
<tr>
<td>30</td>
<td>$4.75</td>
</tr>
</tbody>
</table>
Linear equations come in many different forms. So far in this book, you have used the slope-intercept form. The slope-intercept form gives both the slope and y-intercept of the graph. In this lesson, you will work with linear equations in other forms and convert them into slope-intercept form in order to graph the equations.

**STANDARD FORM:**   \( Ax + By = C \) where \( A \) and \( B \) are not both zero.

**POINT-SLOPE FORM:**   \( y - y_1 = m(x - x_1) \) or \( y = mx + y_1 \)

It is important to know how to change equations into forms you can recognize and use. The key to converting a linear equation into slope-intercept form is to remove all parentheses using the Distributive Property and then isolating the \( y \)-variable.

### Converting STANDARD FORM to SLOPE-INTERCEPT FORM

<table>
<thead>
<tr>
<th>Process</th>
<th>Example: (-3x + 4y = 12)</th>
</tr>
</thead>
</table>
| Step 1: Move the term containing the \( x \)-value to the other side of the = sign with the constant. | \(-3x + 4y = 12\)  
\[ +3x \]  
\[ \frac{4y}{4} \]  
\[ y = \frac{12 + 3x}{4} \]  
\[ y = \frac{3}{4}x + 3 \]  
| Step 2: Re-write the equation. Remember that you cannot combine terms unless they are like terms. |  
| Step 3: Isolate \( y \) by dividing by the coefficient on the \( y \)-value. Divide EVERY TERM by the coefficient of \( y \). |  
| Step 4: Re-write the equation. Leave the coefficient of \( x \) as a simplified fraction when you divide because this is the slope of the graph. |  

### EXAMPLE 1

#### Convert the following from STANDARD form to SLOPE-INTERCEPT form.

**a.** \(-5x + 2y = -20\)

\[
\begin{align*}
2y & = -20 + 5x \\
\frac{2y}{2} & = \frac{-20 + 5x}{2} \\
y & = -10 + \frac{5}{2}x \\
or \quad y & = \frac{5}{2}x - 10
\end{align*}
\]

**b.** \(x - 3y = 9\)

\[
\begin{align*}
-3y & = 9 - x \\
\frac{-3y}{-3} & = \frac{9 - x}{-3} \\
y & = -3 + \frac{1}{3}x \\
or \quad y & = \frac{1}{3}x - 3
\end{align*}
\]

Don't forget there is a 1 in front of the \( x \).
Convert the following from Point-Slope form to Slope-Intercept form.

\[ a. \quad y = \frac{1}{2} (x - 4) + 1 \]

**Solution:**

\[
\begin{align*}
A. \quad y &= \frac{1}{2} (x - 4) + 1 \\
&= \frac{1}{2} x - 2 + 1 \\
&= \frac{1}{2} x - 1
\end{align*}
\]

\[ b. \quad y + 4 = -3(x - 2) \]

**Solution:**

\[
\begin{align*}
b. \quad y + 4 &= -3(x - 2) \\
&= -3x + 6 \\
&= -4 \\
y &= -3x + 2
\end{align*}
\]

In Example 2, the equation \( y = \frac{1}{2} (x - 4) + 1 \) is equivalent to \( y = \frac{1}{2} x - 1 \). That means that both equations represent the same line even though the equations look different. Linear equations can come in many forms, but every linear equation can be converted to an equivalent slope-intercept equation. Try the Explore! to practice converting all types of equations to slope-intercept form.

**Converting Equations to Slope-Intercept Form**

1. Use the Distributive Property to remove any parentheses.
2. Combine all like terms on the same side of the equals sign.
3. Isolate \( y \) by balancing the equation using the Properties of Equality.
4. Re-write the equation in slope-intercept form.

**Explore!**

In each set of four equations “one of these things is not like the other”. Three of the linear equations in each set are equivalent and one is not. For each set, find the three that are similar and give the slope-intercept form that they are equivalent to. Graph that equation on a coordinate plane.

**Set 1**

\[
\begin{align*}
y + 10 &= 3(x + 2) \\
y &= 1 + 3(x + 1) \\
-9x + 3y &= -12 \\
6x - 2y &= 8
\end{align*}
\]

**Set 2**

\[
\begin{align*}
2x + 4y &= 4 \\
y &= \frac{1}{2}(x + 6) + 4 \\
y + 1 &= -\frac{1}{2}(x - 4) \\
5x + 10y &= 10
\end{align*}
\]
EXERCISES

Match each equation to its equivalent equation in slope-intercept form.

1. \( y + 6 = 3(x + 2) \)  
   A. \( y = 4x - 2 \)

2. \( y = \frac{1}{2}(x + 8) - 2 \)  
   B. \( y = \frac{1}{2}x + 2 \)

3. \( y + 1 = 1(x - 3) \)  
   C. \( y = 3x \)

4. \(-4x + y = -2 \)  
   D. \( y = -\frac{1}{2}x + 2 \)

5. \( 2x - 4y = -4 \)  
   E. \( y = x - 4 \)

6. \( 2x + 4y = 8 \)  
   F. \( y = \frac{1}{2}x + 1 \)

Convert each equation to slope-intercept form.

7. \( y + 3 = 4(x + 6) \)  
   8. \( 6x + 2y = 12 \)  
   9. \( y = -2 + \frac{1}{3}(x + 9) \)

10. \( 2x - 5y = -15 \)  
    11. \( -x - 2y = 2 \)  
    12. \( y - 1 = -2(x - 5) \)

13. \( y = \frac{3}{4}(x + 12) - 2 \)  
    14. \( -7x + y = 6 \)  
    15. \( y + 15 = 4(x + 6) \)

One of the two equations listed in each problem matches the graph. Determine which equation is represented by the graph.

16. \( y - 1 = 2(x + 1) \) \hspace{1cm} \text{OR} \hspace{1cm} 6x + 3y = 9

17. \( 6x + 2y = -4 \) \hspace{1cm} \text{OR} \hspace{1cm} y = \frac{1}{3}(x - 9) + 1

REVIEW

Write an equation in slope-intercept form that satisfies the information given about the line.

18. has a slope of \( \frac{5}{2} \) and a \( y \)-intercept of 3

19. has a slope of \(-3\) and goes through the point \((3, 1)\)

20. has a slope of 5 and goes through the origin

21. goes through the points \((6, 1)\) and \((10, -1)\)

22. goes through the points \((-2, 5)\) and \((4, 11)\)

23. has a slope of 0 and a \( y \)-intercept of \(-5\)

24. goes through the points \((4, 1)\) and \((4, 9)\)

25. goes through the points \((1, -3)\) and \((2, -6)\)
Lesson 19 ~ Different Forms Of Linear Equations

**Tic-Tac-Toe ~ Equivalent Equations**

**Step 1:** On a sheet of notebook paper, write 10 linear equations in point-slope or standard form. Next to each equation, write the equivalent linear equation in slope-intercept form.

**Step 2:** Take a blank sheet of white paper and fold it down the middle vertically. Cut the front sheet into equal-sized sections.

**Step 3:** Write a linear equation from your list that is not in slope-intercept form on each flap. Inside the flap, show the steps to reach the equivalent equation that is in slope-intercept form.

**Tic-Tac-Toe ~ Class Competition**

Create a game that can be played with the whole class as a review for the material in this Block. Create a “Teacher’s Guide” with the four categories shown below. In each category, generate three questions that vary in level of difficulty from easiest to hardest. Include the answer for each question in the “Teacher’s Guide.”

<table>
<thead>
<tr>
<th>Writing Equations from Graphs</th>
<th>Writing Equations from Key Information</th>
<th>Graphing Linear Equations</th>
<th>Equivalent Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 1</td>
<td>Level 1</td>
<td>Level 1</td>
<td>Level 1</td>
</tr>
<tr>
<td>Level 2</td>
<td>Level 2</td>
<td>Level 2</td>
<td>Level 2</td>
</tr>
<tr>
<td>Level 3</td>
<td>Level 3</td>
<td>Level 3</td>
<td>Level 3</td>
</tr>
</tbody>
</table>

Create a document explaining the class competition rules. Here are some things to consider:

- How many teams should there be?
- How are the teams decided?
- How do the teams answer the questions (as individuals or as a team)?
- How much time does a team have to answer a question?
- What happens if a team gets the answer wrong?
- How do teams score points?
- When is the game over?
Jared and Wendy are playing a matching game. Each card in the deck has either a graph or a linear equation on it. The goal of the game is to be the first to match the six Equation Cards to their corresponding Graph Cards.

**Step 1:** Convert each Equation Card to slope-intercept form.

**Step 2:** Match each Equation Card to its corresponding Graph Card.

**Step 3:** On your own paper, create two more Equation Cards and their corresponding graph cards that could be used in a future matching game.
No matter what form a linear equation is written in, it can be converted to slope-intercept form and graphed. Remember that the key steps to converting an equation into slope-intercept form include using the Distributive Property and isolating the \( y \)-variable.

**EXAMPLE 1**

Convert the following equations to slope-intercept form and then graph.

a. \( 4x + 3y = 12 \)

\[
\begin{align*}
-4x &
\quad \\
3y &= 12 - 4x \\
y &= 4 - \frac{4}{3}x
\end{align*}
\]

b. \( y = \frac{1}{2}(x - 4) + 3 \)

\[
\begin{align*}
y &= \frac{1}{2}x - 2 + 3 \\
y &= \frac{1}{2}x + 1
\end{align*}
\]

Every line has an infinite number of points that make up the line. In certain situations, verification is needed to determine whether or not a point lies on a certain line. In order to determine this, the equation DOES NOT need to be graphed. The \( x \)- and \( y \)-values from the ordered pair can be substituted for the \( x \)-and \( y \)-variables in the linear equation. If the values make the equation true, then the point lies on the line.

**EXAMPLE 2**

Determine if each point is on the given line.

a. Is the point \((3, -4)\) on the line \(5x + 2y = 7\)?

\[
\begin{align*}
5x + 2y &= 7 \\
5(3) + 2(-4) &= 15 + -8 \\
7 &= 7
\end{align*}
\]

b. Is the point \((0, 5)\) on the line \(y + 3 = 2(x + 3)\)?

\[
\begin{align*}
y + 3 &= 2(x + 3) \\
5 &= 2(0 + 3) \\
8 &= 2(3) \\
8 &\neq 6
\end{align*}
\]
Convert each equation to slope-intercept form and graph. Clearly mark at least three points on each line.

1. \(-5x + 2y = -8\)  
2. \(y - 8 = -3(x + 1)\)  
3. \(y = 1 + \frac{1}{2}(x + 4)\)

4. \(x + 4y = 12\)
5. \(4x - 3y = 6\)
6. \(y = 2(x + 5) - 8\)

7. \(4x = 8\)
8. \(y = \frac{3}{4}(x + 8) - 9\)
9. \(y + 6 = 3(x + 2)\)

10. \(y - 11 = -\frac{5}{2}(x + 2)\)
11. \(-x + y = -6\)
12. \(-5y = 20\)

13. Write a linear equation that is not in slope-intercept form. Convert it to slope-intercept form and graph it.

14. Patti signed up for a cell phone plan that charges an initial monthly fee and a set rate per minute she talks on the phone each month. The equation she was given to calculate her total bill, \(y\), was \(y = 0.1(x + 20) + 7\) where \(x\) represents the number of minutes she talks on the phone in one month.
   a. Convert the equation into slope-intercept form.
   b. What is Patti’s initial fee each month?
   c. What is her rate per minute?
   d. Last month, Patti talked on the phone 427 minutes. How much will her total bill be for last month?

15. Is the point \((-1, 4)\) on the line \(3x + 2y = 5\)?
16. Is the point \((6, 0)\) on the line \(y = -11 + 2(x - 1)\)?

17. Is the point \((0, 0)\) on the line \(y = -\frac{1}{2}(x + 4) + 2\)?
18. Is the point \((2, 10)\) on the line \(5x - y = 0\)?

19. Is the point \((-6, -2)\) on the line \(-x + 2y = -10\)?
20. Is the point \((-\frac{1}{2}, 3)\) on the line \(y = 2 - 2x\)?

21. Vicky decided to buy tickets to the local Razorbacks baseball games. She learned she must first become a Razorback Club member for $14 and then pay $2.50 per ticket to attend the games. The equation that represents the total cost, \(C\), based on the number of games, \(g\), she attends is \(C = 14 + 2.50g\). Vicky purchased 12 games plus the membership fee. The ticket sales person charged her $34. Was she charged correctly? If not, how much should she have been charged?

In each set of three linear equations, two are equivalent. Identify the one linear equation that is not equivalent to the others in the set.

22. \[
\begin{align*}
2x + 4y &= 8 \\
y &= \frac{1}{2}x - 2 \\
y &= -\frac{1}{2}x + 2
\end{align*}
\]
23. \[
\begin{align*}
y - 3x &= 4 \\
y &= 3(x - 2) + 2 \\
-3x + y &= 4
\end{align*}
\]
24. \[
\begin{align*}
y &= x - 1 \\
5x + 5y &= -5 \\
y &= -(x + 6) + 5
\end{align*}
\]
Write an equation in slope-intercept form that is represented by the given information.

25. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

26. has a slope of \(-2\) and \(y\)-intercept of 1

27. 

28. goes through the points \((-3, 3)\) and \((2, 8)\)

29. 

30. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>34</td>
</tr>
</tbody>
</table>

31. 

32. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

33. has a slope of \(\frac{1}{3}\) and goes through the point \((6, 1)\)

---

**Tic-Tac-Toe ~ Careers Using Algebra**

Linear equations are an essential part of Algebra I. There are many career choices where knowledge of algebra is crucial. Research at least two different careers that require the use of algebra. Write a 1-2 page report about these careers.

Include the following in your report for each career:
- Description of the career
- How the career includes the use of algebra
- How much schooling is required for the career
Exponential equations are used to predict growth and decay. Exponential equations have a start value and multiplication as the repeated operation. The start value is represented by the variable \( b \) and the constant multiplier is represented by the variable \( m \) in the equation \( y = b \cdot m^x \).

Most growth and decay situations involve percentages. For example, a car may depreciate (decrease in value) at a rate of 12% per year. Or the number of bacteria might increase by 40% each day. In order to find the value of \( m \) with percentages, you must first start with 100% and then add or subtract the given percentage depending on whether it is increasing or decreasing in value. The percent must be converted to a decimal.

Car depreciating by 12% = 100% − 12% = 88% = 0.88
Bacteria increasing by 40% = 100% + 40% = 140% = 1.40

A car was purchased for $14,000 and depreciates at a rate of 12% per year. This means it keeps 88% of its value. The exponential equation would be:

\[ y = 14,000 \cdot 0.88^x \]

You can find the new value of the car after any number of years by substituting the number of years for \( x \). For example, to find the value of the car after 5 years, substitute 5 for \( x \). Remember to follow the order of operations when calculating.

\[ y = 14,000 \cdot 0.88^5 \approx 7,388.25 \]

**Answer each of the following exercises using the exponential equation \( y = b \cdot m^x \).**

1. Mary bought a brand new car in 2004 for $22,000. She was told that her model of car has an annual depreciation rate of 11%.
   a. How much was Mary’s car worth if she sold it in 2007?
   b. How much will her car be worth if she waits and sells it in 2015?

2. Imar made $4,000 over the summer and wants to invest it for the future. He finds a bank that offers him 6% growth each year.
   a. How much will Imar have in the account after 3 years?
   b. How much will he have after 8 years?
   c. After approximately how many years will he have doubled his money?

3. The number of bacteria in a kitchen sink increases by 60% each day the sink remains unclean. The sink currently has 15 bacteria.
   a. How many bacteria will be present after 10 days of not washing the sink?
   b. Approximately how long will it take before there are at least 1,000 bacteria in the sink?

4. Write two of your own growth or decay application problems and find the solutions.
In this textbook you have learned how to graph linear functions when written in different forms. However, not all functions produce a linear graph. In this lesson you will learn about a few types of non-linear functions. Non-linear functions are functions that, when graphed, do not form a line. There are a large variety of non-linear functions, many of which you will learn about in higher-level mathematics.

In a linear function, each equal “step” that the $x$-value increases, the $y$-value increases or decreases a consistent amount.

In a non-linear function, there is not a consistent adding or subtracting pattern for each equal “step”, although each type of non-linear function does have a unique pattern. In this lesson you will examine quadratic functions, exponential functions and inverse variation functions. Each type of non-linear function has a parent graph. The parent graphs are the most basic graph of each non-linear function.

**EXPLORE!**

### The Quadratic Function

**Step 1:** The parent graph of the quadratic function is the graph of $y = x^2$. Copy the table at right on your own paper and fill in the missing values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
</tr>
</tbody>
</table>

**Step 2:** Plot the $(x, y)$ points from the table. Connect the points with a curved line. Describe what the graph looks like.

### The Exponential Function

**Step 3:** One parent graph of an exponential function is the graph of $y = 2^x$. Copy the table at right on your own paper and fill in the missing values using a calculator.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3$</td>
<td>$2^{-3} = 0.125$</td>
</tr>
<tr>
<td>$-2$</td>
<td>$2^{-2} = 0.25$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$2^{-1} = 0.5$</td>
</tr>
</tbody>
</table>

**Step 4:** Plot the $(x, y)$ points from the table. Connect the points with a curved line. Describe what the graph looks like.
The parent graphs of three non-linear functions are shown below. Other graphs in each “family” have the same shape as the parent graph but may be stretched, shrunk, moved or flipped.

**Quadratic Functions**

\[ y = x^2 \]

**Exponential Functions**

\[ y = 2^x \]

**Inverse Variation Functions**

\[ y = \frac{1}{x} \]

**Example 1**

Determine if each graph, table or equation is linear or non-linear. If it is non-linear, identify the type of function (quadratic, exponential or inverse variation).

a. \( y = 3x - 2 \)

b. 

![Graph](image)

b. NON-LINEAR. This graph forms a “U” shape so it is a quadratic function.

c. 

![Graph](image)

c. NON-LINEAR. The graph matches the inverse variation parent graph.

d. 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>( \frac{1}{3} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
</tr>
</tbody>
</table>

d. NON-LINEAR. Graph the data points to see that it matches the parent function graph of the exponential function.

**Solutions**

a. LINEAR. This equation is linear because it is in slope-intercept form.

b. NON-LINEAR. This graph forms a “U” shape so it is a quadratic function.

c. NON-LINEAR. The graph matches the inverse variation parent graph.

d. NON-LINEAR. Graph the data points to see that it matches the parent function graph of the exponential function.
Non-linear equations are used in the real world. Banks use exponential functions to calculate interest. Architects use quadratic functions when designing bridges. Businessmen use quadratic functions to determine costs of products in order to maximize profits. Chemists use exponential functions to calculate the rate of bacteria growth. An example of a real-world use of the inverse variation function is found in the creation of levers.

EXERCISES

Copy each table and use the given equation to fill in the missing values.

1. \[ y = x^2 + 2 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>((-2)^2 + 2 = 6)</td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

2. \[ y = 3^x \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>(3^{-2} = \frac{1}{9})</td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

3. \[ y = 2x - 5 \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>(2(-2) - 5 = -9)</td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

4. Graph the five points from the table in Exercise 1 on a coordinate plane. What type of non-linear equation is this?

5. Graph the five points from the table in Exercise 2 on a coordinate plane. What type of non-linear equation is this?

6. Graph the five points from the table in Exercise 3 on a coordinate plane. What type of equation is this?

Determine if each graph, table or equation is linear or non-linear. If it is non-linear, identify the type of graph (quadratic, exponential or inverse variation).

7. \[ x \quad y \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>(\frac{1}{16})</td>
</tr>
<tr>
<td>-1</td>
<td>(\frac{1}{4})</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
</tr>
</tbody>
</table>

8. \[ y = (x + 1)^2 \]

9. \[ y = 4^x \]

10. [Graph from text]

11. \[ x \quad y \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
</tbody>
</table>
13. A bacteria culture begins with 5 cells and doubles every hour. This situation can be represented by the exponential function \( y = 5 \cdot 2^x \) where \( x \) represents the number of hours and \( y \) represents the number of bacteria. How many bacteria will be in the culture after 7 hours?

14. The path of a ball thrown through the air can be modeled by the equation \( y = -4.9x^2 + 17x + 3.4 \) when \( x \) is the time in seconds and \( y \) is the height of the ball in meters. Find the height of the ball after 3 seconds.

**REVIEW**

Solve each equation. Check your solution.

15. \( x - 24 = 72 \)  
16. \( \frac{x}{5} = -11 \)  
17. \( 8x + 13 = 61 \)

18. \( 17 = -4x + 2 \)

19. \( 2x + 7 = 5x - 8 \)

20. \( 3(x - 5) = -9 \)

Draw a coordinate plane for each problem and graph the given equation. Clearly mark three points on the line.

21. \( y = \frac{1}{3}x - 3 \)

22. \( y = 1 \)

23. \( y = 2x + 1 \)

24. \( y = x \)

25. \( y = -\frac{5}{2}x + 4 \)

26. \( y = -4x + 9 \)

27. \( x = -1 \)

28. \( y = -\frac{1}{2}(x - 6) - 1 \)

29. \( 2x + 5y = 10 \)
Tic-Tac-Toe ~ Quadratic Functions

Quadratic functions can be described as being “U” shaped. Linear equations can be easily graphed when in slope-intercept form. Similarly, quadratic functions can be easily graphed when in factored form: $y = (x - a)(x - b)$. You need to know the two $x$-intercepts and the vertex (maximum or minimum point on the graph) in order to graph a simple quadratic function.

For example: Graph $y = (x - 4)(x + 2)$.

**Step 1:** Locate the $x$-intercepts. The $x$-intercepts can be found by setting the expressions inside each parentheses equal to 0 and solving.

\[
\begin{align*}
  x - 4 &= 0 \\
  +4 &+4
\end{align*}
\]
\[
\begin{align*}
  x + 2 &= 0 \\
  -2 &-2
\end{align*}
\]

\[
x = 4 \\
x = -2
\]

**Step 2:** Average the $x$-intercepts by adding them together and then dividing the sum by 2. This is the $x$-coordinate of the vertex.

\[
\frac{4 + (-2)}{2} = 1
\]

**Step 3:** Substitute the number from **Step 2** back into the original equation to find the $y$-coordinate of the vertex.

\[
y = (1 - 4)(1 + 2)
\]
\[
y = (-3)(3) = -9
\]

**Step 4:** Graph the quadratic function. Connect the three points with a smooth curve.

Graph the following quadratic functions following the four steps shown above.

1. $y = (x - 2)(x - 6)$
2. $y = (x - 5)(x + 1)$
3. $y = (x - 2)(x + 2)$
4. $y = (x + 5)(x + 1)$
Lesson 16 ~ Graphing Using Slope-Intercept Form

Draw a coordinate plane for each problem and graph the given equation. Clearly mark three points on the line.

1. \( y = 3x - 4 \)
2. \( y = \frac{2}{3}x + 3 \)
3. \( y = x - 1 \)
4. \( y = -2x \)
5. \( x = 3 \)
6. \( y = 6 + \frac{4}{3}x \)

7. Create a linear equation that satisfies each condition. Graph your equations on a coordinate plane.
   a. Slope = 2 and a negative \( y \)-intercept
   b. Slope = 0 and a \( y \)-intercept of \( -3 \)
   c. A negative slope and a positive \( y \)-intercept
   d. A positive slope and a \( y \)-intercept of 0

Lesson 17 ~ Writing Linear Equations for Graphs

Identify the slope and \( y \)-intercept of each graph and write the corresponding linear equation in slope-intercept form.

8.

9.

10.
11. At two different times during the summer, Leticia measured the height of a tomato plant she had planted in June. She measured it when she first planted it and then again 4 weeks later.
   a. Find the slope-intercept equation that represents the height of Leticia's tomato plant based on the number of weeks since she planted it.
   b. Use your equation to determine exactly how tall the tomato plant will be after 7 weeks.
   c. Use your equation to determine how many weeks have passed if the plant is 29 inches tall.

Lesson 18 ~ Writing Linear Equations from Key Information

Write an equation in slope-intercept form when given key information about a line.

12. slope = $\frac{3}{4}$, $y$-intercept = 5
13. slope = $-5$, $y$-intercept = 1
14. slope = 1, $y$-intercept = 9
15. slope = $\frac{2}{5}$, $y$-intercept = 0
16. slope = 2, goes through the point (2, 3)
17. slope = $\frac{1}{2}$, goes through the point (6, 1)
18. slope = $-1$, goes through the point $(-3, 5)$
19. slope = $\frac{5}{2}$, goes through the point $(-6, -10)$
20. goes through the points (1, 1) and (5, 9)
21. goes through the points $(-6, 0)$ and (3, 3)
22. goes through the points $(-4, 8)$ and $(-3, 5)$
23. goes through the points (8, $-4$) and (5, $-4$)
24. A furniture rental company rents large screen televisions. They charge an initial fee plus $20 for each day the TV is rented. Steven rented the TV for 8 days and was charged $225. Let $x$ represent the number of days and $y$ represent the total cost of the rental.
   a. Identify the slope and one ordered pair from the information given.
   b. Find the equation of the line that fits this information.
   c. If another customer rents the TV for 17 days, how much should he expect to pay?
25. A canoe rental company on Deep Sea Lake rents canoes for a set fee plus an additional charge per hour. Marshall asked two different individuals how many hours they had rented their canoes for and how much it cost. One rented a canoe for 4 hours and paid $32. Another person rented a canoe for 10 hours for $56. Let $x$ represent the length of time in hours and let $y$ represent the total cost.
   a. Write two ordered pairs that use the data Marshall collected.
   b. Find the equation of the line that goes through these two points.
   c. What number in the linear equation represents the amount of the set fee?
   d. What is the real world meaning of the slope in this equation?
   e. How much will someone pay for a canoe rental from this company if she keeps the canoe for 6 hours?
Lesson 19 ~ Different Forms of Linear Equations

Convert each equation to slope-intercept form.

26. \(y = 4 + 2(x - 7)\)  
27. \(3x + 6y = 18\)  
28. \(y = \frac{1}{4}(x + 4) - 3\)  
29. \(4x - 5y = 15\)  
30. \(-x + 3y = -12\)  
31. \(y - 2 = 3(x + 1)\)

Lesson 20 ~ More Graphing Linear Equations

Convert each equation to slope-intercept form and graph. Clearly mark at least three points on each line.

32. \(-4x + 2y = -6\)  
33. \(y + 1 = 3(x - 2)\)  
34. \(y = \frac{3}{2}(x - 4) + 2\)  
35. \(7x = -14\)  
36. \(x + 3y = 12\)  
37. \(y = 2(x - 1) + 2\)

Determine if each point is on the given line.

38. Is the point \((-2, 1)\) on the line \(4x - 3y = -11\)?
39. Is the point \((2, 5)\) on the line \(y = 2(x - 1) + 3\)?
40. Is the point \((-6, 0)\) on the line \(y + 4 = \frac{1}{2}(x + 4) + 3\)?

Lesson 21 ~ Introduction to Non-Linear Functions

Determine if each graph, table or equation is linear or non-linear. If it is non-linear, identify the type of graph (quadratic, exponential or inverse variation).

41. 
<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>12</td>
</tr>
<tr>
<td>-1</td>
<td>3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
</tbody>
</table>

42. \(y = \frac{2}{3}x - 4\)

43. 

44. 

45. 
<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>

46.
Scott
Retail Manager
Dallas, Oregon

I am a retail manager for a department store. My position plays an important part in making sure that our department makes money. Many decisions that I make affect how much money our department brings in. I try to maximize our profits by making sure that our products are good and are priced right. I also determine how many employees to hire and how many should work each day. If too many people are working, profits get spent on labor we do not need.

I use math every day in my job as a retail manager. I use basic math skills in my job as well as complex equations. Math helps me to determine how to get the most profits for the department. The profit of my department is affected by “shrink.” Shrink is a term that we use in retail to describe theft, damaged or broken products, and products that we never received but still get billed for. Profits are also affected by the cost of transportation, electricity, building maintenance and advertising. As a retail manager, I can control how many employees we are using and also reduce shrink. Both of these things will improve the company’s total profits.

I was hired as a retail manager after I had completed my college degree. A person can get hired in my career with a high school diploma, but will need to go through lots of on the job training. People usually have to work their way up through the company to get into a management position.

Salaries for retail managers can vary quite a bit. Salaries start as low as minimum wage for people without experience or other training. People who get into a management training program can earn around $25,000 per year. After becoming a manager, salaries range from $40,000 to $60,000 per year.

I like my job as a retail manager because I enjoy working with the public. I also like how fast things change in the retail world. The best part of my job, though, is helping new employees turn into great long-term members of our team.