## BLOCK 1 ~ TWO - DIMENSIONAL GEOMETRY

### Tic - Tac - Toe

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The word geometry comes from two Greek words: *geo* meaning earth and *metron* meaning measure. Geometry includes relationships of points, lines, angles, surfaces and solids. You will be examining two- and three-dimensional geometry in this book.

**Area** is the number of square units needed to cover a space. Look at the rectangle below. The base is 4 units. The height is 2 units. There are 8 square units covering the space inside the rectangle. The area of the rectangle is 8 square units.

![Rectangle](image)

In previous years, you have learned how to find areas of four basic geometric shapes. The area of a shape is found by substituting information about a shape into the appropriate area formula.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
</tr>
<tr>
<td>Rectangle</td>
<td>$A = lw$</td>
</tr>
<tr>
<td>Square</td>
<td>$A = lw$ or $s^2$</td>
</tr>
<tr>
<td>Parallelogram</td>
<td>$A = bh$</td>
</tr>
</tbody>
</table>

**EXAMPLE 1**

Find the area of the triangle.

**Solution**

Write the formula. $A = \frac{1}{2}bh$

Substitute the given information. Area = \frac{1}{2}(8)(5)

Multiply. Area = 20

The area of the triangle is 20 square centimeters.

You can also work backwards to find a missing measure on a figure using the formula. Substitute all known values into the formula. Solve the equation for the unknown measure.
Finding a Missing Measure

1. Find the formula which fits the given figure.
2. Substitute all known values for the variables in the formula.
3. Determine if any numbers on the same side of the equals sign can be combined. If so, combine those numbers.
4. Write the answer in a complete sentence.

Example 2

Find each missing measure.

a. $A = 72 \ m^2$

{\[
\text{Area} = lw \\
72 = 12x \\
\frac{72}{12} = \frac{12x}{12} \\
6 = x
\]}

The width of the rectangle is 6 meters.

b. $5h^2 = 3$

{\[
\text{Area} = \frac{1}{2}bh \\
35 = \frac{1}{2}(y)(7) \\
35 = 3.5y \\
\frac{35}{3.5} = \frac{3.5y}{3.5} \\
y = 10
\]}

The base of the triangle is 10 inches.

**Perimeter** is the distance around the outside of a figure. You will need to be able to find the perimeter of figures. You may also need to use your knowledge of perimeter to find the area of a figure.
Lesson 1 ~ Areas Of Triangles And Parallelograms

EXAMPLE 3

The perimeter of a square bathroom tile is 32 centimeters. Find the area of the tile.

**Solution**

Each side of a square is the same length. Divide the perimeter by 4 to find the length of one side.

\[32 \div 4 = 8\]

Write the formula for the area of a square. 

\[\text{Area} = s^2\]

Substitute the values for the variables.

\[\text{Area} = 8^2\]

\[\text{Area} = 64\]

The area of the bathroom tile is 64 square centimeters.

**Exercises**

Find the area of each figure.

1. 

2. 

3. 

4. 

5. 

6. 

Sketch a diagram of each figure and label it with the given information. Calculate the area of each figure.

7. A triangle has a base of 8 cm and a height of 5 cm.

8. The base of a parallelogram is 21 feet and the height is 9 feet.

9. The width of a rectangle is 4.5 inches. The length is 3.5 inches longer than the width.

Find the missing measure.

10. \[A = 20 \text{ in}^2\]

11. \[A = 42 \text{ cm}^2\]

12. \[A = 45 \text{ square units}\]
13. $A = 81 \text{ m}^2$

14. $A = 52 \text{ ft}^2$

15. $A = 19.5 \text{ m}^2$

16. The perimeter of a square is 72 m.
   a. What is the length of one side?
   b. Find the area of the square.

17. The length of a rectangle is 2.5 cm. The area is 20 cm$^2$. What is the width of the rectangle?

18. A triangle has a height of 11 ft and an area of 110 ft$^2$. What is the length of the base?

19. Lois built a rectangular flower bed. She used four boards for the border of the flower bed. Two of the boards were 4 feet long. The other boards were 8 feet long.
   a. What was the area of the flower bed using these boards?
   b. She decided to cut each board in half to make two square flower beds. What was the area of each square flower bed?
   c. Did one or two flower beds give Lois more total area to plant flowers?

20. A football field is 120 yards long and 160 feet wide.
   a. Convert the length of the football field from yards to feet.
      (1 yard = 3 feet)
   b. What is the area of a football field in square feet?

21. A window had 8 panes. Each pane measured 8 inches by 10 inches. What is the area of the entire window?

Plot each set of points on a coordinate plane. Connect the points in the order given. Find the area of each figure.

22. (3, 5), (3, 1) and (9, 1)

23. (0, −4), (0, 0), (−4, 0) and (−4, −4)

24. (−2, 5), (−2, −1), (4, −1) and (4, 5)

25. (0, 3), (5, 3), (4, −1) and (−1, −1)

26. Find the perimeter of each figure in Exercises 23 and 24.

27. Estimate the perimeter of the figure in Exercise 25. Are you able to find the exact perimeter? Why or why not?

28. On the Oregon State Assessment formula sheet, the formula for area of a rectangle is given but not the formula for the area of a square. Why do you think this is the case?
Formulas are used for common geometric figures. What happens if something is not a common geometric shape? Estimation is important to give an idea of a figure's area. You will estimate the area of your foot in this activity.

Step 1: Trace your foot on a piece of grid paper.
Step 2: Estimate how many square units your foot is. Each square on the paper represents one square unit.

Step 3: Look at the shape of your foot. Piece together geometric shapes (squares, triangles, parallelograms and rectangles) that cover your foot outline.
Step 4: Calculate the area of each figure on a piece of paper. Find the sum of these areas.
Step 5: Compare your estimated answer with the sum of the areas of geometric shapes.
Step 6: Write two to three paragraphs discussing your observations, conclusions and thoughts on which method for calculating the area of your foot was best.

Quadrilaterals are geometric figures that have four sides. You have worked with squares, rectangles, trapezoids and parallelograms. Two additional types of quadrilaterals are the kite and the rhombus.

1. Find and record the definition of a kite and a rhombus. Draw an example of each.

2. List the special properties of a kite and a rhombus.

3. Find or develop the formulas used to calculate areas of a kite and a rhombus.

4. Copy each shape below onto your own paper. Find the area of each shape. Show all work.

5. Draw two additional kites and two additional rhombi. Measure the key dimensions on each to the nearest tenth of a centimeter. Find the area of each figure.
A trapezoid is a quadrilateral with two parallel sides. The figures below are trapezoids.

The parallel sides of a trapezoid are called the bases. A trapezoid has two bases. Each is identified using a subscript number. Subscript numbers identify objects that have the same name but represent different parts. The height of a trapezoid is the length between the bases.

**EXPLORE!**

**Step 1:** Trace the trapezoid shown. Label $b_1$, $b_2$ and $h$.

**Step 2:** Two trapezoids can be connected to form a parallelogram. Trace the trapezoid a second time so it connects with the first one, forming a parallelogram. *(Hint: you may have to turn your paper.)* Label $b_1$, $b_2$ and $h$ on the second trapezoid.

**Step 3:** Write an expression for the base of the parallelogram.

**Step 4:** Write an expression for the area of the parallelogram.

**Step 5:** How does the area of one trapezoid compare to the area of the parallelogram? Rewrite the expression from **Step 4** to represent the area of one of the trapezoids.

**Step 6:** Use the formula from **Step 5** to calculate the area of each trapezoid below.

**A FORMULA FOR TRAPEZOID AREA**

$$A = \frac{1}{2}h(b_1 + b_2)$$
**EXAMPLE 1**

Find the area of the trapezoid.

![Trapezoid Diagram]

**Solution**

Write the trapezoid area formula. 
\[
\text{Area} = \frac{1}{2} h(b_1 + b_2)
\]

Substitute all known values. 
\[
\text{Area} = \frac{1}{2} (4)(5 + 9)
\]

Add the numbers in parentheses. 
\[
\text{Area} = \frac{1}{2} (4)(14)
\]

Multiply. 
\[
\text{Area} = 2(14) = 28
\]

The area of the trapezoid is 28 ft².

A missing measure on a trapezoid can be found by substituting the known values into the formula. Once all the known values have been substituted, you can solve the equation for the missing measure.

**EXAMPLE 2**

Find the height of the trapezoid. The trapezoid’s area is 17.5 cm².

![Trapezoid Diagram]

**Solution**

Write the trapezoid area formula. 
\[
\text{Area} = \frac{1}{2} h(b_1 + b_2)
\]

Substitute all known values. 
\[
17.5 = \frac{1}{2} h(8 + 6)
\]

Add the numbers in parentheses. 
\[
17.5 = \frac{1}{2} h(14)
\]

Multiply \(\frac{1}{2}\) and 14. 
\[
17.5 = 7h
\]

Divide by 7 on both sides of the equation. 
\[
\frac{17.5}{7} = \frac{7h}{7}
\]

\[
2.5 = h
\]

The height of the trapezoid is 2.5 cm.
Example 3

Nakisha painted a large trapezoid-shaped mural. The area of the mural is 88 square feet. The longer base is 20 feet. The height is 6.4 feet. Find the length of the missing base.

Solution

Draw a figure to represent the given information.

Write the trapezoid area formula.

Substitute all known values.

Multiply \( \frac{1}{2} \) and 6.4.

Distribute 3.2.

Subtract 64 from both sides of the equation.

Divide by 3.2.

The length of the missing base on the mural is 7.5 feet.

Exercises

Calculate the area of each trapezoid.

1. 

2. Height = 3 m

3. 

4. 

5. 

6. 

7. Find the area of a trapezoid with the following measures: \( b_1 = 24 \) inches, \( b_2 = 30 \) inches, \( h = 6 \) inches

8. A trapezoid has a height of 12.3 miles. The bases measure 3.5 miles and 14.5 miles. Find the area.

9. The bases of a trapezoid measure 6 feet and 14 feet. The height of the trapezoid is 7 feet. Find the area.
10. Celeste used trapezoidal bricks for a patio. Each brick has base measures of 20 cm and 35 cm. The distance between the bases (height) is 12 cm.
   a. What is the area of one brick?
   b. It took 812 bricks to cover the patio. What was the total area of the patio?

Find the area of each figure.

11. \begin{align*}
    \text{Area} &= \frac{1}{2}(b_1 + b_2) \times h \\
    &= \frac{1}{2}(5\text{ in} + 18\text{ in}) \times 10\text{ in} \\
    &= \frac{1}{2} \times 23\text{ in} \times 10\text{ in} \\
    &= 115\text{ in}^2
\end{align*}

12. \begin{align*}
    \text{Area} &= \frac{1}{2}(b_1 + b_2) \times h \\
    &= \frac{1}{2}(10\text{ cm} + 5\text{ cm}) \times 6\text{ cm} \\
    &= \frac{1}{2} \times 15\text{ cm} \times 6\text{ cm} \\
    &= 45\text{ cm}^2
\end{align*}

Find the unknown base or height of each trapezoid.

13. \( A = 60 \text{ ft}^2 \)  
    \begin{align*}
    4\text{ ft} \\ h \\ 8\text{ ft}
    \end{align*}

14. \( A = 77 \text{ in}^2 \)  
    \begin{align*}
    9\text{ in} \\ 7\text{ in} \\
    \frac{9\text{ in} + 7\text{ in}}{2} = 8\text{ in}
    \end{align*}

15. \( A = 27.03 \text{ m}^2 \)  
    \begin{align*}
    b_1 = 5.1\text{ m} \\ b_2 = 2.4\text{ m} \\
    A = \frac{1}{2}(5.1\text{ m} + 2.4\text{ m}) \times h \\
    27.03 = \frac{1}{2}(7.5\text{ m}) \times h \\
    h = \frac{2\times 27.03}{7.5} = 7.33\text{ m}
    \end{align*}

16. A trapezoid has an area of 62 square inches. The bases measure 8 and 12 inches. Find the height.

17. Find the missing base measurement for a trapezoid with the following measures:
   \( b_1 = 9 \text{ meters}, h = 6 \text{ meters} \) and \( A = 51 \text{ m}^2 \)

18. The area of a trapezoid is 102 square feet. The height is 12 feet. One of the bases is 8 feet long. Find the missing measure.

19. The area of a trapezoid is 100 square inches. The height is 10 inches. Give two pairs of possible lengths for the bases.

20. The top of a trapezoidal photo frame is 10 inches. The base measures 18 inches. The area of the frame is 224 square inches. Find the height of the frame.

21. Use the green trapezoid.
   a. Find the area of the top triangle (A).
   b. Find the area of the bottom triangle (B).
   c. Find the sum of the triangles’ area.
   d. Use the trapezoid area formula to calculate the area. Does your answer match part c?
22. Use the blue trapezoid.
   a. Find the area of the triangle on the left.
   b. Find the area of the rectangle in the middle.
   c. Find the area of the triangle on the right.
   d. Add the answers from a-c to find the area of the trapezoid.
   e. Find the length of \( b_1 \) and \( b_2 \).
   f. Calculate the area of the trapezoid using the trapezoid area formula.
   g. Compare and contrast the two different methods used to find the area of the trapezoid. Describe a time when finding the area of each piece may be more helpful than using the formula.

23. Use the figure to the right.
   a. Find the area of the red and green trapezoid.
   b. Add the areas together.
   c. What shape is the entire figure?
   d. Calculate the area of the entire figure using the formula \( A = bh \).
   e. Are the answers in parts b and d the same? Why?

**REVIEW**

Find the area of each figure.

24.  

25. \( 7 \text{ m} \)

26. 

Find the missing measure.

27. \( A = 90 \text{ in}^2 \)

28. \( A = 25 \text{ square units} \)

29. \( A = 44 \text{ m}^2 \)

**Tic-Tac-Toe ~ Flashcards**

Create a set of 30 flashcards to help a student review the material in Block 1. Use cardstock paper or blank index cards. Write the question on one side of the card and the answer on the other side. The set should include at least three cards from each of the following categories:

- Areas of basic geometric figures
- Areas of composite figures
- Vocabulary
- Word problems
- Uses of the different estimates of \( \pi \)
- Parts of a circle
A circle is the set of all points that are the same distance from a point called the center. A circle is named using its center point. The circle to the right is named \( \odot A \), which is read “circle A.”

<table>
<thead>
<tr>
<th>Vocabulary</th>
<th>Diagram</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td><img src="image" alt="Circle with a radius" /></td>
<td>The distance from the center of a circle to any point on the circle.</td>
</tr>
<tr>
<td>Chord</td>
<td><img src="image" alt="Circle with chords" /></td>
<td>A line segment with endpoints on the circle.</td>
</tr>
<tr>
<td>Diameter</td>
<td><img src="image" alt="Circle with a diameter" /></td>
<td>The distance across a circle through the center. The diameter is the longest chord. It is made of 2 radii.</td>
</tr>
<tr>
<td>Central Angle</td>
<td><img src="image" alt="Circle with a central angle" /></td>
<td>An angle with its vertex at the center of the circle.</td>
</tr>
</tbody>
</table>

Circles can be drawn using a stencil, tracing around a circular object or using a compass. You can also sketch a circle by drawing points an equal distance from the center and connecting the points with a smooth curve.
In geometry, a line segment is named using its two endpoints.

An angle is formed by two rays with a common endpoint. Angles are often named using a letter on each ray and a letter at the common endpoint. The order the letters are written in the name must follow the order they appear in the angle.

Diameters, radii, chords and central angles are named by the points on the circles.

**EXAMPLE 1**

Use \( \odot C \) to name each circle part.

a. A chord
b. A radius
c. A diameter
d. A central angle

**Solutions**

a. Any segment with endpoints on the circle. \( \overline{AB} \) or \( \overline{EF} \)
b. A segment from the center of the circle to any point on the circle. \( \overline{CD} \) or \( \overline{CE} \) or \( \overline{CF} \)
c. A chord that goes through the center. \( \overline{EF} \)
d. An angle with the vertex at the center of the circle. \( \angle ECD \), \( \angle FCD \) or \( \angle FCE \)

A diameter is a special chord. It has endpoints on the circle, but it also goes through the center of the circle. It is twice as long as a radius. The diameter is the distance across a circle through its center.

For a diameter \( d \), and its radius, \( r \):

\[ d = 2r \quad r = \frac{1}{2}d \]

**EXAMPLE 2**

a. The diameter of a circle is 12 centimeters. Find the radius.
b. The radius of a circle is 7 inches. Find the diameter.

**Solutions**

a. A radius is half the length of a diameter.

Find the radius by dividing the diameter by 2. \( 12 \div 2 = 6 \)
The radius is 6 centimeters.

b. A diameter is twice as long as a radius.

Find the diameter by multiplying the radius by 2. \( 7 \cdot 2 = 14 \)
The diameter is 14 inches.

A central angle in a circle is an angle with its vertex at the center of the circle. It is measured using degrees. Each circle has 360° around the center point. There are two central angles in the circle to the left.


### Lesson 3 ~ Parts Of A Circle

#### Example 3

**Find the measure of the missing central angle.**

**Solution**

The sum of the four central angles is 360°.

Write an equation. \[140 + 40 + 100 + x = 360\]

Combine like terms. \[280 + x = 360\]

Subtract 280 from both sides of the equation.

\[x = 80\]

The missing angle, x, is 80°.

### Exercises

1. List at least 5 circular objects seen or used in everyday life.

2. What are three ways to draw a circle?

3. Draw \(\odot K\) with radius \(\overline{PK}\).

4. Draw a circle with chord \(\overline{AB}\).

5. Draw a circle with radius \(\overline{HM}\) and diameter \(\overline{MX}\).

6. Draw \(\odot R\) with central angle \(\angle NRE\).

7. Draw a circle with a radius of 3 centimeters. Explain the process you used to draw the circle.

8. Draw a circle with a diameter of 2 inches.

**Name each of the following for \(\odot B\).**

9. A radius  

10. A diameter  

11. A central angle  

12. A chord  

**Name each of the following for \(\odot V\).**

13. three radii  

14. two diameters  

15. two chords  

16. three central angles
Find the diameter of a circle with each radius.

17. 28 m  
18. 6.25 yd  
19. $4\frac{2}{3}$ in

Find the radius of a circle with each diameter.

20. 8 cm  
21. 15 yd  
22. 9.4 mm

Find the missing measure of each unknown central angle.

23.  
24.  
25.  
26.  
27.  
28.  

29. A circle has 4 central angles. Three of the angles measure 70° each. What is the measure of the fourth angle?

30. $\odot W$ is divided in half. There are three central angles on one half of the circle. Two of the central angles measure 43° and 76°. What is the measure of the third central angle on that half of the circle?

**REVIEW**

Calculate the area of each figure.

31.  
32.  
33.  
34.  
35.  
36.  

16 Lesson 3 ~ Parts Of A Circle
The distance around an object is called the perimeter. The distance around a circle is called the circumference. There is a special relationship between the circumference of a circle and the diameter.

**EXPLORE!**

**Step 1:** Gather 5 circular objects (such as lids, cups, cans, etc) to measure.

**Step 2:** Copy the chart.

<table>
<thead>
<tr>
<th>Circular Object</th>
<th>Circumference</th>
<th>Diameter</th>
<th>( \frac{\text{circumference}}{\text{diameter}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</table>

**Step 3:** Write the name of each object in the column “Circular Object”.

**Step 4:** Use a tape measure to find the circumference of each object to the nearest tenth of a centimeter. Record the measurement in the “Circumference” column in the chart.

**Step 5:** Use a tape measure or ruler to find the diameter of each object to the nearest tenth of a centimeter. Record the measurement in the “Diameter” column.

**Step 6:** Find the ratio of the circumference to the diameter for each object. Use a calculator. Write each answer in the last column as a decimal rounded to the nearest hundredth.

**Step 7:** What do you notice about the decimal values in the last column of the chart? Compare with a classmate.

**Step 8:** About how many times larger is the circumference than the diameter? Write a formula for finding the approximate circumference of a circle. \( C = \_ \cdot d \)

**Step 9:** Use the formula to find the approximate circumference of each circle.

a. \[ \text{C} = \frac{5}{2} \cdot \pi \]

b. \[ \text{C} = \frac{3}{2} \cdot \pi \]

c. \[ \text{C} = \frac{7}{2} \cdot \pi \]
The circumference of a circle is a little bit more than three times the length of the diameter. You can wrap 3 diameters along the edge of a circle. There will still be a little bit of the circle not covered because the circumference is larger.

The exact number of times the diameter can be wrapped around the circle is represented by the Greek letter \( \pi \) (pi). Pi is the ratio of the circumference of a circle to its diameter.

The exact value of \( \pi \) cannot be written as a decimal because it never terminates and never repeats. Most people round \( \pi \) to the nearest hundredth and use the number 3.14 or the fraction \( \frac{22}{7} \) to estimate \( \pi \).

\[
\text{Circumference of a Circle}
\]

\[ C = \pi d = 2\pi r \]

The circumference of a circle is the product of \( \pi \) and the diameter of the circle.

**Example 1**

Find the circumference of each circle. Use 3.14 for \( \pi \).

**a.**

![Circle H with diameter 8 mm]

**Solutions**

a. Write the circumference formula using the diameter. 
\[ C = \pi d \]
Substitute the known values. 
\[ C \approx 3.14(8) \]
Multiply. 
\[ C \approx 25.12 \]
The circumference of \( \odot H \) is about 25.12 mm.

**b.**

![Circle J with radius 2.5 ft]

Write the circumference formula using the radius. 
\[ C = 2\pi r \]
Substitute the known values. 
\[ C \approx 2(3.14)(2.5) \]
Multiply. 
\[ C \approx 15.7 \]
The circumference of \( \odot J \) is about 15.7 feet.

**Example 2**

Natalya walked with her friend around a circular track. The circumference of the track was 188.4 meters. Find the approximate length of the track’s radius. Use 3.14 for \( \pi \).

**Solution**

Write the circumference formula. 
\[ C = 2\pi r \]
Substitute the known values. 
\[ 188.4 = 2(3.14)r \]
Multiply. 
\[ 188.4 = 6.28r \]
Divide both sides by 6.28. 
\[ \frac{188.4}{6.28} = \frac{6.28r}{6.28} \]
\[ 30 = r \]
The radius of the track was approximately 30 meters.
EXERCISES

Find the circumference of each circle. Use 3.14 for π.

1. \[ C = 12.56 \text{ ft} \]
2. \[ C = 94.2 \text{ cm} \]
3. \[ C = 213.52 \text{ in} \]
4. \[ C = 10 \text{ yd} \]
5. \[ C = 3 \text{ cm} \]
6. \[ C = 3.5 \text{ ft} \]
7. A van manufacturer recommends a wheel with a diameter of 17 inches. What is the circumference of this wheel?
8. A circular doughnut has a radius of 2 inches. Find the circumference.
9. Hubert drove to Pendleton. He noticed a circular irrigation system watering a field of alfalfa near Umatilla. The arm of the sprinkler was 1,250 feet long. Find the distance around the outside of the watered region.
10. The International Space Station orbits approximately 342 kilometers above the earth. The radius of Earth is 6,357 kilometers.
   a. Find the radius of the Space Station’s orbit from the center of the earth.
   b. Find the distance the Space Station travels in one orbit.

Find each missing measure. Use 3.14 for π.

11. \[ C = 12.56 \text{ ft} \]
    \[ d = ? \]
12. \[ C = 94.2 \text{ cm} \]
    \[ r = ? \]
13. \[ C = 213.52 \text{ in} \]
    \[ d = ? \]
14. The circumference of a circle is 16.328 meters. Find the length of the diameter.
15. The circumference of a circle is 62.8 miles. What is the length of the radius?
16. Find the length of the diameter of a circle whose circumference is 29.516 yards.
17. Julio paddled his boat around the edge of a circular lake. He traveled a total of 3.14 miles. What is the diameter of the lake?

18. The center circle on a basketball court has a circumference of 37.68 feet. Find the approximate diameter and radius of the circle at center court.

19. A tractor travels 169.5 inches on one rear tire revolution. What is the diameter of the tire? Round to the nearest whole number.

20. Which has the greater perimeter: a square with side lengths of 10 or a circle with a diameter of 10? Support your answer mathematically.

21. Why are the solutions in Exercises 1-19 approximations?

**REVIEW**

22. Sketch a circle with radius \( \overline{AB} \) and a chord \( \overline{AC} \).

23. Sketch a circle with diameter \( \overline{WY} \) and central angle \( \angle YMP \) which measures 90°.

24. Sketch a triangle with a height of 1.5 inches and base of 2 inches. Find the area of the triangle.

25. A triangle has an area of 42 square meters. The height of the triangle is 14 meters. What is the length of the base?

26. Sketch a trapezoid with bases of 3 and 5 centimeters and a height of 2 centimeters. Find the area of the trapezoid.

27. A trapezoid has an area of 94.5 square inches. The sum of the two bases is 21 inches. What is the height of the trapezoid?

**Tic-Tac-Toe ~ A Song about Pi**

Write a song about the number pi to a tune that is familiar to most people. Your song must have at least two verses and a chorus. Some ideas to include in your song are the different estimates of pi, the uses of pi, the formulas which use pi or the history of the number pi. Neatly write or type the lyrics of your song.
Maria and Greg are making circular coasters for a family dinner. They want the radius of each coaster to be 5 centimeters long. How many square centimeters will each coaster cover on a table?

Maria drew a coordinate plane. She sketched the circular coaster. She estimated the area by counting the number of squares inside the circle. She counted approximately 76 squares or parts of squares inside the drawing. Greg thought there might be a more accurate method to find the area of each coaster.

You will learn how to find the area of a circle using a formula in this lesson.

EXPLORE!

Step 1: Trace a circle onto a piece of paper or use a compass to draw a circle. The radius of the circle should be at least 2 inches.

Step 2: Fold the circle in half four times to get 8 equal-sized parts.

Step 3: Cut the eight equal-sized parts of the circle on the fold lines.

Step 4: Arrange the pieces as shown.

Step 5: Which quadrilateral does the new figure resemble?

Step 6: Fill in the blanks.
   a. The height of the figure is equal to the ______________ of the circle.
   b. The base of the figure is approximately half the ______________ of the circle.

Step 7: How can the area of this figure be found? Create a formula to find the area of the circle.

Step 8: Use the formula to find the area of one coaster described at the beginning of this lesson.
The area formula of a circle can be determined by arranging pieces of a circle to form a parallelogram. The height of the parallelogram is the radius of the circle. The length of the base is half the circumference of the circle.

The area of a parallelogram is found by multiplying base times height. When the circle is arranged to form a parallelogram, the area of the circle can be determined by multiplying the base times the height.

\[ \text{Area} = \text{base} \cdot \text{height} \]

\[ \text{Area} = \frac{1}{2} \cdot C \cdot r \]

\[ \text{Area} = \frac{1}{2} (2\pi r) \cdot r \]

\[ \text{Area} = \pi r \cdot r \]

\[ \text{Area} = \pi r^2 \]

**Example 1**

**Find the area of the circle. Use 3.14 for \( \pi \).**

**Solution**

Write the area formula. \[ A = \pi r^2 \]

Find the length of the radius. \[ r = 10 \div 2 = 5 \]

Substitute the known values. \[ A = 3.14(5)^2 \]

Square 5. \[ A = (3.14)(25) \]

Multiply. \[ A = 78.5 \]

The area of \( \odot A \) is approximately 78.5 square centimeters.

The circle in Example 1 is the same size as Maria’s and Greg’s coasters from the beginning of the Lesson. Each coaster will be approximately 78.5 square centimeters.
Jaira is having pizza for her birthday party. She can make two small pizzas, each with a radius of 6 inches, or one large pizza with a radius of 12 inches. Which option will give her more square inches of pizza?

**Solution**

<table>
<thead>
<tr>
<th>Two small pizzas</th>
<th>One large pizza</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = \pi r^2 )</td>
<td>( A = \pi r^2 )</td>
</tr>
<tr>
<td>( A \approx (3.14)(6)^2 )</td>
<td>( A \approx (3.14)(12)^2 )</td>
</tr>
<tr>
<td>( A \approx (3.14)(36) )</td>
<td>( A \approx (3.14)(144) )</td>
</tr>
<tr>
<td>( A \approx 113.04 )</td>
<td>( A \approx 452.16 )</td>
</tr>
</tbody>
</table>

Multiply the area of a small pizza by 2 for 2 small pizzas.

\[ A \approx 113.04(2) \approx 226.08 \]

The large pizza would give Jaira twice as much pizza as two small pizzas.

Up to this point you have used a common estimate for \( \pi \), 3.14, for calculations. There will be situations when exact answers are needed for problems involving circles. For example, a manufacturer might program a machine to create a circular tarp with a radius of 3 feet. The exact area of one tarp needs to be calculated in order to find the amount of material used for one tarp. Estimating means the answer is not exact.

Exact answers are written using the \( \pi \) symbol. A common estimate of \( \pi \) should not be substituted for \( \pi \) when an exact answer is required.

**Example 3**

Find the exact area of \( \odot W \).

**Solution**

Write the circle area formula.

\[ A = \pi r^2 \]

Substitute known values.

\[ A = \pi(3)^2 \]

Square 3.

\[ A = \pi(9) \]

Rewrite with the number before the symbol.

\[ A = 9\pi \]

The exact area of \( \odot W \) is \( 9\pi \) square feet.
Find the area of each circle. Use 3.14 for π.

1. [Circle with 4 ft radius]
2. [Circle with 22 in radius]
3. [Circle with 10 mm radius]

4. Mark set up a sprinkler to water the backyard. The sprinkler waters in a circular motion. The radius of the area covered by the sprinkler is 12 feet. Find the approximate square footage of the watered area.

5. An extra large pizza has a radius of 16 inches. Find the area of the pizza.

6. A circular tablecloth has a diameter of 14 feet. What is the area of the tablecloth?

7. Heceta Head Lighthouse is located just south of Florence in Southern Oregon. The lighthouse beam reaches 21 miles in all directions. How many square miles does the light cover? Round the answer to the nearest square mile.

8. The diameter of a classroom clock face is 13 inches. Find the area of the clock face.

9. Lacey plans to put another window in her bedroom. She needs to decide whether to put in two small windows, each with a 1 foot radius, or one large window with a 2 foot radius.
   a. Find the total area of the two small windows.
   b. Find the area of the large window.
   c. She wants as much sunlight as possible in her room. Which option should she use?

Calculate the exact area of each circle.

10. [Circle with 5 mm radius]
11. [Circle with 18 in radius]
12. A circle with a diameter of 14.4 feet.

13. A milling machine is programmed to cut a hole with a radius of 8 millimeters. The computer program requires the programmer to enter the area of the hole. Find the exact area of the hole.

14. A circle has a radius of 1 inch. Find the exact area of the circle.
15. A dart board has a circumference of 56.52 inches.
   a. Find the diameter of the dartboard. Use 3.14 for $\pi$.
   b. Find the approximate area of the dart board.

16. The exact circumference of a circle is 6$\pi$ feet.
   a. What is the length of the radius?
   b. Find the exact area of the circle.

17. The circumference of a circle is 7.85 meters.
   a. What is the length of the radius? Use 3.14 for $\pi$.
   b. Find the area of the circle. Round the answer to the nearest hundredth.

18. The area of a circle is 16$\pi$ in$^2$. What is the diameter?

19. Each circle has a radius of 4 cm. Find the exact area of each shaded region, given that the regions in each circle are equal in size.
   a. 
   b. 
   c. 
   d. 

**REVIEW**

Use the diagram for Exercises 20 - 25.

20. Name the center.  
21. Identify two radii.  
22. Identify a central angle.  
23. Name the shortest chord.  
24. Name the longest chord.  
25. What is the name of the circle?

**Tic-Tac-Toe ~ Ferris Wheels**

Write a research paper about the Ferris wheel. It must be at least one page in length. Include the following:

- Facts about the first Ferris wheel
- Information about 2 additional Ferris wheels
- Measurements for diameter, radius and circumference
- A list of resources used to find the information
- A diagram of one Ferris wheel you researched labeled with the measurements
You have found exact answers for circumference and area using π. You have also used one common estimate of pi, 3.14. The fraction \( \frac{22}{7} \) is another approximation of pi. There are actually three common approximations of pi. You can also use the π button on a calculator.

### EXPLORE!

**Step 1:** Copy and complete the table below. Use each approximation of pi to find the circumference and area of \( \odot A \).

<table>
<thead>
<tr>
<th>Approximation of Pi</th>
<th>Radius of Circle</th>
<th>Circumference (2πr)</th>
<th>Area (πr²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.14</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{22}{7} )</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Calculator π</td>
<td>7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 2:** Most calculators show 3.141592654 as the value of π. Why is this considered an approximation?

**Step 3:** Which of the above approximations gives the least accurate answer? How do you know?

**Step 4:** Which of the above approximations gives the most accurate answer? How do you know?

**Step 5:** List the approximations of π from least to greatest.

**Step 6:** The most common estimate of π is 3.14. Why do you think it is the most commonly used estimate?

**Step 7:** For what type of numbers might \( \frac{22}{7} \) be the most useful form to use for π?
EXAMPLE 1

Use the $\pi$ button on your calculator to find the circumference and area of $\odot H$. Round to the nearest hundredth.

SOLUTION

<table>
<thead>
<tr>
<th>Circumference</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = 2\pi r$</td>
<td>$A = \pi r^2$</td>
</tr>
</tbody>
</table>

Substitute all known values.

- $C = 2(\pi)(4.2)$
- $A = \pi(4.2)^2$

Simplify.

- $(8.4)\pi$
- $(4.2)^2$

Multiply using $\pi$ button.

- $26.3937829$
- $55.41769441$

Round to the nearest hundredth.

- $26.39$
- $55.42$

The circumference of $\odot H$ is about $26.39\text{ cm}$ and the area of $\odot H$ is about $55.42\text{ cm}^2$.

EXAMPLE 2

Find the area and circumference a circle with a 28 inch diameter. Use $\frac{22}{7}$ for $\pi$.

SOLUTION

<table>
<thead>
<tr>
<th>Circumference</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C = 2\pi r$</td>
<td>$A = \pi r^2$</td>
</tr>
</tbody>
</table>

Substitute all known values.

- $C \approx 2\left(\frac{22}{7}\right)(14)$
- $A \approx \left(\frac{22}{7}\right)(14)^2$

Rewrite with fractions.

- $\approx \left(\frac{2}{7}\right)\left(\frac{22}{7}\right)\left(\frac{14}{1}\right)$
- $\approx \left(\frac{22}{7}\right)\left(\frac{14}{1}\right)\left(\frac{14}{1}\right)$

Multiply.

- $\approx 616$\div 7 \approx 88$
- $\approx 4312$\div 7 \approx 616$

The circumference of the circle is about 88 inches. The circle area is about 616 $\text{in}^2$.

CHOOSEING COMMON ESTIMATES OF PI

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Used as a very rough estimate of $\pi$.</td>
</tr>
<tr>
<td>3.14</td>
<td>Used often as the most common estimate of $\pi$.</td>
</tr>
<tr>
<td>$\frac{22}{7}$</td>
<td>Best used when the radius is a multiple of 7 or if the radius is a fraction.</td>
</tr>
<tr>
<td>Calculator $\pi$</td>
<td>The most accurate estimate. Used when a calculator is available.</td>
</tr>
</tbody>
</table>

EXAMPLE 3

Choose the most appropriate value of $\pi$ for each description. Explain why you made that choice.

a. Find the area of a circle with a radius of 14.

b. Find the circumference of a circle when you want a rough estimate.

c. Find the most accurate area of a circle using an approximation.

SOLUTIONS

a. The most appropriate value of $\pi$ is $\frac{22}{7}$ since 14 is a multiple of 7.

b. Three can be used for $\pi$ when only needing a rough estimate.

c. The $\pi$ button on the calculator should be used when the most accurate estimate is needed.
Identify the most appropriate form of pi to use in each situation. Explain your choice.

1. Habika orders bark dust for a circular flower bed. She is calculating the area of the flower bed.

2. Marlon programs a computer to make circular parts for dental equipment.

3. Hiroshi finds the circumference of a circle with a radius of 42 feet.

4. Gayle roughly estimates the circumference of her circular driveway.

Use a calculator π button to find the circumference and area of each circle. Round to the nearest hundredth.

5. \( 4 \text{ cm} \)

6. \( 7.6 \text{ cm} \)

7. \( \odot P \) with diameter 12.5 cm

8. \( \odot M \) with radius of 2\( \frac{1}{2} \) ft

Find the circumference and area of each circle in Exercises 9-13. Use \( \frac{22}{7} \) for \( \pi \).

9. A circle with a radius of 14 feet.

10. A circle with a diameter of 70 meters.

11. \( \text{7 yards} \)

12. \( 3.5 \text{ cm} \)

13. \( 21 \text{ miles} \)

Choose an appropriate value of pi to answer each exercise.

14. Find a rough estimate of the area of a circular field with a radius of 110 feet.

15. Kekona knows the radius of her circular driveway is 20 feet. She will border it with decorative stone. Find a rough estimate of the circumference of Kekona’s driveway.

16. Pizza comes in 12 inch or 14 inch diameters. Graysen really likes the crust. How much more crust will Graysen get if he orders a 14 inch pizza instead of a 12 inch pizza, to the nearest inch?

Use 3.14, \( \frac{22}{7} \), the calculator π button or exact terms to answer each question. Round to the nearest hundredth as needed.

17. A twirled lasso has a radius of 6 feet. What is the area enclosed by the lasso’s circle?

18. What is the radius of a circle given \( C = 113.04 \text{ m} \)?
19. $\odot M$ has a circumference of $14\pi$ centimeters. What is the area of $\odot M$?

20. The first Ferris wheel had a diameter of 76 meters.
   a. How far did a person travel in one revolution?
   b. How far in 10 revolutions?

21. A circular pool has a diameter of 21 feet. The pool cover has an overhang of one foot. Find the circumference of the cover.

**REVIEW**

Write each area formula using variables. Identify the shape or shapes which use each formula.

22. base times height
23. one-half base times height
24. radius squared times $\pi$
25. one half the height times the quantity of base one plus base two
26. length times width
27. length of a side squared

**Tic-Tac-Toe ~ Equal Areas**

1. Draw a square that has an area of 36 square inches. Cut out the figure and label the lengths of the sides inside the square.

2. Create six more figures that all have areas of 36 square inches. The figures must include a triangle, a rectangle, a parallelogram, a trapezoid, a circle and a composite figure. Cut out each figure and label the key dimensions on each figure.

3. Make a poster displaying each geometric shape, its dimensions and its area calculation.

**Tic-Tac-Toe ~ Greeting Cards**

1. Design eight greeting cards. The set should include cards made of each of the following shapes:
   - Triangle
   - Square
   - Parallelogram
   - 2 Composite Figures
   - Rectangle
   - Trapezoid
   - Circle

2. Measure the key dimensions on each shape to the nearest tenth of a centimeter. Find the area of each cut-out and record it along with the dimensions on the back of the greeting card.

3. Illustrate or color each card. Write a different greeting or expression on each card.

4. Package the cards creatively as if they will be displayed and sold at a store.
Composite figures are made up of two or more geometric shapes. Figures joined with shapes at their edges or with a part(s) removed are composite figures. Below is a summary of the formulas you have learned to this point.

- **Triangle**
  \[ A = \frac{1}{2}bh \]

- **Rectangle**
  \[ A = lw \]

- **Square**
  \[ A = lw \text{ or } s \cdot s \]

- **Parallelogram**
  \[ A = bh \]

- **Trapezoid**
  \[ A = \frac{1}{2}h(b_1 + b_2) \]

- **Circle**
  \[ A = \pi r^2 \]
  \[ C = 2\pi r \]

The area of each shape in a composite figure must be determined in order to find the total area of the figure. Once the areas are calculated, decide whether to add or subtract the areas to find the overall area of the composite figure.

Find the total area of the figure shown below by breaking it up into familiar shapes. Add the areas of the shapes together.

Find the shaded area of the figure below by calculating the area of each figure. Subtract the smaller area from the larger area.
Lesson 7 – Composite Figures

**EXAMPLE 1**

Calculate the area of the shaded region.

![Diagram of shaded region]

**Solution**

Draw a diagram.

Find the area of each shape.

- \[ \text{Area} = lw = 8 \cdot 4 = 32 \text{ m}^2 \]
- \[ \text{Area} = \frac{1}{2}bh = \frac{1}{2}(8)(3) = 12 \text{ m}^2 \]
- \[ \text{Area} = lw = 5 \cdot 3 = 15 \text{ m}^2 \]

Add the areas of the three figures.

\[ 32 + 12 + 15 = 59 \text{ m}^2 \]

The area of the shaded region is 59 square meters.

**EXAMPLE 2**

Calculate the area of the shaded region. Use 3.14 for \( \pi \).

![Diagram of shaded region]

**Solution**

Draw a diagram.

Find the area of the rectangle.

\[ \text{Area} = lw = 10 \cdot 30 = 300 \text{ cm}^2 \]

The length of the rectangle is three diameters and the width is one diameter.

Find the area of one circle.

\[ \text{Area} = \pi r^2 = (3.14)(5)^2 \approx 78.5 \text{ cm}^2 \]

Subtract the area of the three circles from the area of the rectangle.

\[ 300 - 78.5 - 78.5 - 78.5 = 64.5 \text{ cm}^2 \]

The area of the shaded region is about 64.5 cm².
Use a diagram to show how to find the area of each shaded region.

1. 

2. 

3. 

4. 

5. 

6. 

Calculate the area of each shaded region. Use 3.14 for π. If necessary, round to the nearest hundredth.

7. 

8. 

9. AB = 4 cm, AC = 6 cm

10. 

11. 

12. 

13. 

14. 

15. d = 42 mm
Find the distance around each figure (perimeter). Use 3.14 for π.

16.\[
\begin{array}{c}
\text{6 cm} \\
\text{4 cm} \\
\text{3.5 cm} \\
\text{13 cm}
\end{array}
\]

17.\[
\begin{array}{c}
\text{6 ft} \\
\text{14 ft}
\end{array}
\]

18.\[
\begin{array}{c}
\text{7 in}
\end{array}
\]


20. The bases of a trapezoid are 13 cm and 8 cm. The height is 6 cm. Amir found the largest triangle that will fit inside the trapezoid.
   a. What were the dimensions for the triangle?
   b. He cut the triangle out of the trapezoid. How much area remained after the triangle was removed?

21. Sydney needs to cut rectangles from a 4 by 8 foot sheet of plywood. Each rectangle she cuts needs to be 1 foot by 3 feet. What is the maximum number of rectangles she can cut?

22. A rectangular track surrounds the edge of the football field at McKinley Middle School. The football field is 360 feet long and 160 feet wide. The inside edge of the 10 foot wide track borders the football field.
   a. Draw a diagram of the track and the football field.
   b. Label dimensions for the football field and track.
   c. Find the total area of the track.

23. Claire sent a postcard to her grandmother. The card was 3 inches by 5 inches. She put a circular sticker on one side. The sticker had a radius of 1.25 inches. How much space is left on the side with the sticker for her to write on? Use 3.14 for π. Round to the nearest hundredth.

REVIEW

Find the degree measure of each missing central angle.

24.\[
\begin{array}{c}
\text{41°} \\
\text{x} \\
\text{139°} \\
\text{81°}
\end{array}
\]

25.\[
\begin{array}{c}
\text{90°} \\
\text{x} \\
\text{25°}
\end{array}
\]

26.\[
\begin{array}{c}
\text{58°} \\
\text{x} \\
\text{122°} \\
\text{58°}
\end{array}
\]

Plot each set of points. Connect in the order given. Find the area of each figure.

27. (3, 5), (3, -2), (-4, -2)

28. (0, -3), (8, -3), (8, 1), (0, 1)
Two figures with the exact same shape, but not necessarily the same size, are called similar figures. All circles have the exact same shape; therefore, all circles are similar. Similar figures can be compared using a ratio. A ratio is a comparison of two numbers using division. The ratio of \( a \) to \( b \) can be written three different ways.

\[
\frac{a}{b} \quad a : b \quad a \text{ to } b
\]

**EXPLORE!**

Latrelle is putting stepping stones around his backyard. He bought three different sizes of circular stepping stones. He wants to compare the three sizes with ratios.

**Step 1:** Write a ratio comparing the radius of one circle to the radius of another circle. Write each ratio as a fraction in simplest form.

- a. \( \odot A \) to \( \odot C \)
- b. \( \odot B \) to \( \odot C \)
- c. \( \odot A \) to \( \odot B \)

**Step 2:** Find the exact circumference of each circle.

**Step 3:** Write a ratio comparing the exact circumference of one circle to the circumference of another circle. Write each ratio as a fraction in simplest form.

- a. \( \odot A \) to \( \odot C \)
- b. \( \odot B \) to \( \odot C \)
- c. \( \odot A \) to \( \odot B \)

\[
\frac{2\pi}{3\pi} \text{ simplifies to } \frac{2}{3}
\]

**Step 4:** What do you notice about the ratios in Step 1 compared to the ratios in Step 3?

**Step 5:** What do you predict about the ratios of the diameters for the three stepping stones?

**Step 6:** Latrelle buys one extra-large stepping stone with a circumference of \( 48\pi \) inches. How many times larger is this stepping stone’s radius than that of \( \odot C \)? Explain your answer.

**Step 7:** Find the exact area of each of Latrelle’s stepping stones.

**Step 8:** Write a ratio comparing the area of one circle to the area of another circle. Write each ratio as a fraction in simplest form.

- a. \( \odot A \) to \( \odot C \)
- b. \( \odot B \) to \( \odot C \)
- c. \( \odot A \) to \( \odot B \)
Lesson 8 ~ Circle Similarity

Step 9: What do you notice about the ratios in Step 8 compared to the original ratios in Step 1?

Step 10: Suppose Latrelle’s one extra-large stepping stone has a radius 6 times as large as the radius of his smallest stone. How many times larger is the area of his extra-large stone compared to his smallest stone?

Circle Similarity

If the ratio of the radii of two circles is \(a:b\), then:
- The ratio of their diameters is \(a:b\).
- The ratio of their circumferences is \(a:b\).
- The ratio of their areas is \(a^2:b^2\).

Example 1

Two circles have diameters of 12 cm and 4 cm. What is the ratio of their circumferences?

Solution

Write a ratio comparing the diameters. \(\frac{12 \text{ cm}}{4 \text{ cm}}\)

Simplify. \(\frac{12 \text{ cm}}{4 \text{ cm}} = \frac{3}{1}\)

The circumferences have the same ratio as the diameters. The ratio of their circumferences is 3 : 1.

Example 2

Two circles have circumferences of 24\(\pi\) and 36\(\pi\). Find the ratio of their areas.

Solution

Write a ratio comparing their circumferences. \(\frac{24\pi}{36\pi}\)

Simplify. \(\frac{24\pi}{36\pi} = \frac{24}{36} = \frac{2}{3}\)

Square the ratio to find the ratio of their areas. \(\frac{2}{3} \rightarrow \frac{2^2}{3^2} = \frac{4}{9}\)

The ratio of their areas is 4 : 9.

A proportion is an equation stating two ratios are equivalent. Proportions can be used to find missing measures between two circles. Proportions can be solved using cross products.
EXAMPLE 3

Use a proportion to find the length of the radius in \( \odot Y \).

\[ C = 78.5 \text{ in} \quad C = 15.7 \text{ in} \]

Write a ratio comparing the circumferences.

\[ \frac{78.5}{15.7} \]

Write a ratio comparing the radii.

\[ \frac{12.5}{r} \]

Set the two ratios equal to each other.

\[ \frac{78.5}{12.5} = \frac{15.7}{r} \]

Set the cross products equal to each other.

\[ (12.5)(15.7) = 78.5r \]

Multiply, then divide on both sides.

\[ \frac{196.25}{78.5} = \frac{78.5r}{78.5} \]

\[ 2.5 = r \]

The radius of \( \odot Y \) is 2.5 in.

EXERCISES

Write a ratio comparing the radii of each pair of circles. Write the ratio in simplest form.

1. 

1. \[ \frac{25 \text{ m}}{5 \text{ m}} \]

2. \( C = 24\pi, C = 6\pi \)

3. 

4. \( \odot P \) has a diameter of 52 yards. \( \odot W \) has a diameter of 52 yards. Write a ratio comparing the circles’ radii.

5. The radius of \( \odot A \) is 4 in. The radius of \( \odot B \) is 22 in. Write a ratio comparing the circles’ circumferences.

6. The circumference of one circle is 100 miles. The circumference of another circle is 75 miles. Write a ratio comparing the circles’ diameters.

7. A car tire has a diameter of 30.4 inches. A tractor tire has a diameter of 76 inches. Write a ratio comparing the circumferences of the tires. Write the ratio as a fraction without decimals and in simplest form.
Write a ratio comparing the areas of each pair of circles. Write the ratio in simplest form.

8. \[ \frac{3}{5} \]  
9. \( C = 100 \text{ cm}, \ C = 400 \text{ cm} \)  
10. \( \frac{2}{9} \)

11. \( \odot A \) has a diameter of 32 feet. \( \odot M \) has a diameter of 24 feet. Write a ratio comparing the circles’ areas.

Use a proportion to find each missing measure.

12. \( C = 50.24 \text{ units} \)  
13. \( C = 56.52 \text{ m} \)  
14. \( C = 6.28 \text{ units} \)  
15. \( C = 37.68 \text{ in} \)

14. The circumference of \( \odot A \) is 100.48 inches. The diameter is 32 inches. The circumference of \( \odot B \) is 113.04 inches. Find the diameter of \( \odot B \).

15. The radii of two circles have a ratio of 2 : 5. The area of the larger circle is 125 square meters. Find the area of the smaller circle.

16. The diameters of two circles have a ratio of 3 : 4. The area of the smaller circle is 162 square units. Find the area of the larger circle.

17. The circumference of one circle is five times as large as the circumference of another circle. The radius of the smaller circle is 7.1 cm. Find the length of the larger circle’s radius.

18. Jackson is shopping for a circular swimming pool. The Water Mania pool has a radius of 10 feet. The circumference is 62.8 feet. The circumference of the Splash Attack pool is 81.64 feet.
   a. Find the radius of the Splash Attack pool.
   b. Jackson’s backyard has room for a pool with a radius of 12 feet. Jackson tells his mother the Splash Attack pool will fit perfectly. Do you agree or disagree? Explain your answer.

19. Explain why all circles are similar.
Find the area of each shaded region. Use 3.14 for π. Round to the nearest hundredth, if necessary.

20. \[ \text{Area} = \frac{1}{2} \times \pi \times (18 \text{ cm})^2 \]

21. \[ \text{Area} = \pi \times (7 \text{ ft})^2 \]

22. \[ \text{Area} = \pi \times (14 \text{ cm})^2 \]

23. \[ \text{Area} = 3 \times \pi \times (7 \text{ cm})^2 \]

24. \[ \text{Area} = \pi \times (3 \text{ cm})^2 - \pi \times (2 \text{ cm})^2 \]

**Tic-Tac-Toe ~ Circles and Squares**

1. Find the area of the shaded region in each figure. The length of each side of the square is 10 cm. Use 3.14 for π.

   a. [Circled Diagram]
   b. [Quadrant Diagrams]
   c. [Nested Circles Diagrams]

2. How many circles will be in each of the next two figures if the pattern is continued?

3. Sketch diagrams of the next two figures. Shade to match figures a, b and c.

4. Calculate the area of the “shaded” region in your new diagrams.

5. What percentage of the area is shaded in each figure? Do you see a pattern? Explain.
A sector is a portion of a circle contained by two radii. A sector is sometimes referred to as a slice of a circle. A sector is described by the measure of its central angle. Remember that the sum of all central angles in a circle is 360°.

⊙C has a sector that is 90°. Ninety degrees is a fraction of the whole circle, which is 360°. This ratio can be used in a proportion to find the area of the sector. The ratio of the central angle to 360° can be set equal to the ratio of the area of the sector to the area of the circle.

\[
\frac{90°}{360°} = \frac{\text{area of sector}}{\text{area of } \odot C}
\]

**Area of a Sector**

\[
\frac{\text{degree of the sector}}{360°} = \frac{\text{area of the sector}}{\text{area of the circle}}
\]

**Example 1**
The area of ⊙A is 60 cm². Use a proportion to find the area of the sector.

**Solution**

Write a proportion.

\[
\frac{90°}{360°} = \frac{x}{60}
\]

Set the cross products equal to each other.

\[
90(60) = 360x
\]

Multiply.

\[
5400 = 360x
\]

Divide both sides of the equation by 360.

\[
\frac{5400}{360} = \frac{360x}{360}
\]

\[
15 = x
\]

The area of the sector is 15 cm².
EXAMPLE 2

The Macy family bought a family-sized chocolate chip cookie for dessert. Erin ate a slice of the cookie with a central angle of 37°. The entire cookie had an area of 249 square centimeters. Find the area of Erin's slice to the nearest hundredth.

SOLUTION

Write a proportion. \[
\frac{37^\circ}{360^\circ} = \frac{x}{249}
\]

Set the cross products equal to each other. \[37(249) = 360x\]

Multiply. \[9213 = 360x\]

Divide both sides of the equation by 360. \[\frac{9213}{360} = \frac{360x}{360}\]

\[25.59 \approx x\]

The area of Erin's slice of cookie was approximately 25.59 square centimeters.

EXAMPLE 3

Find the area of the shaded sector in \(\bigodot M\). Round to the nearest hundredth.

SOLUTION

Find the area of the circle. \[\text{Area} = \pi r^2\]
\[\text{Area} = (3.14)(4)^2\]
\[\text{Area} \approx 50.24 \text{ square inches}\]

Substitute all known values into a proportion. \[\frac{115^\circ}{360^\circ} = \frac{x}{50.24}\]

Set the cross products equal to each other. \[(115)(50.24) \approx 360x\]

Multiply and divide to find \(x\). \[\frac{5777.6}{360} = \frac{360x}{360}\]

\[16.048 \approx x\]

Round to the nearest hundredth. \[16.05 \approx x\]

The area of the shaded sector is approximately 16.05 square inches.
Find the area of each shaded sector. Round answers to the nearest hundredth, when necessary.

1. \( A = 50 \text{ cm}^2 \)

2. \( A = 28 \text{ in}^2 \)

3. \( A = 46.5 \text{ ft}^2 \)

4. The area of a circle is 50.24 square miles. What is the area of a sector with a 60° central angle?

5. The radius of a circle is 3 inches. Find the area of a sector with a 35° central angle. Use 3.14 for \( \pi \).

6. A sprinkler rotates back and forth watering a sector with a 75° central angle. The sprinkler has a 50 foot radius. Find the area the sprinkler waters. Use 3.14 for \( \pi \).

7. The hour hand on a living room clock is 4 inches long. Use 3.14 for \( \pi \).
   a. What is the central angle of the hour hand from noon to 9 p.m.?
   b. What is the area of the clock that is crossed over by the hour hand from noon to 9 p.m.?

Use a proportion to find the area of each shaded region. Use 3.14 for \( \pi \). Round to the nearest hundredth, when necessary.

8. 

9. 

10. 

11. 

12. 

13. 

14. Determine in which situation Isaiah will get the most to eat.
   **Situation 1:** A pizza cut into 8 pieces has a diameter of 14 inches. Isaiah eats 5 pieces.
   **Situation 2:** A pizza with a diameter of 18 inches is cut into 12 pieces. Isaiah eats 4 pieces.

15. The area of a circle is 36\( \pi \text{ yd}^2 \). Find the exact area of a sector with a central angle of 30°.

16. A circle with a radius of 100 kilometers has a sector with a central angle of 10°. Find the area of the sector.
17. A family ate all but one slice of pie. The remaining slice of pie has a central angle of 60°. The pie pan has a diameter of 8 inches. Find the area of the pie the family ate.

18. Find the area of sector 1 in $\odot B$.

19. Find the area of sector 1 in $\odot T$.

20. LaTisha’s swimming pool has a circumference of 31.4 feet. What is the length of the radius of the pool?

21. The circumferences of two circles have a ratio of 2 : 3. The diameter of the larger circle is 7.5 units. Find the diameter of the smaller circle.

22. The radii of two circles have a ratio of 2 : 5. The area of the smaller circle is 350 meters.
   a. Find the ratio of the areas of the circles.
   b. Find the area of the larger circle.

Tic-Tac-Toe ~ Pie Charts

A pie chart is a data display using sectors of a circle. The pie chart below shows the results of a survey of high school students about their favorite subject in school.

1. Use a proportion or the percent equation to determine the measure of each central angle in the pie chart. Remember that the sum of all central angles in a circle is 360°.

2. Measure the length of the radius of the pie chart using centimeters. Find the area of each sector. Use 3.14 for $\pi$. Round to the nearest hundredth. Label each answer based on the subject that sector represents on the chart.

3. Another pie chart has an area of 314 square centimeters. Find the central angles of each of the four regions given the area of their sectors.

Favorite Sport to Watch

<table>
<thead>
<tr>
<th>Sport</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Football</td>
<td>131.88 cm²</td>
</tr>
<tr>
<td>Basketball</td>
<td>87.92 cm²</td>
</tr>
<tr>
<td>Baseball</td>
<td>53.38 cm²</td>
</tr>
<tr>
<td>Soccer</td>
<td>40.82 cm²</td>
</tr>
</tbody>
</table>
Lesson 1 ~ Areas of Triangles and Parallelograms

Find the area of each figure.

1. \[ A = \frac{1}{2} \times 5 \text{ ft} \times 11 \text{ ft} \]

2. \[ A = \frac{1}{2} \times 4 \text{ cm} \times 9 \text{ cm} \]

3. \[ A = \frac{1}{2} \times 7 \text{ yd} \times 7 \text{ yd} \]

4. Jacklyn has a triangle pendant with a base of 9 mm and a height of 5 mm. What is the area of the pendant?

5. A square has a perimeter of 24 feet. What is the area of the square?

Find each missing measure.

6. \[ A = 30 \text{ ft}^2 \]

7. \[ A = 18 \text{ m}^2 \]

8. \[ A = 23.4 \text{ in}^2 \]
Lesson 2 ~ Area of a Trapezoid

Find the area of each trapezoid.

9. \[ \text{Base measures are 5.7 cm and 8.3 cm. The height of the trapezoid is 9 cm.} \]

Find the unknown base or height of each trapezoid.

12. \[ A = 45 \text{ square inches} \]
\[ b_1 = 7 \text{ inches} \]
\[ b_2 = 11 \text{ inches} \]
\[ h = ? \]

13. \[ A = 160 \text{ in}^2 \]
\[ \text{base measures} \]
\[ 6 \text{ in} \]
\[ 8 \text{ in} \]
\[ b_1 \]

14. \[ A = 76 \text{ m}^2 \]
\[ \text{base measures} \]
\[ 5.2 \text{ m} \]
\[ 7.6 \text{ m} \]
\[ b_1 \]

15. A brick of gold looks like a trapezoid from a side view. The length of the top is 3.5 inches. The bottom base measures 7 inches. The area of a side of the trapezoid is approximately 17.5 square inches. What is the height?

Lesson 3 ~ Parts of a Circle

Use \( \odot H \) to name each of the following.

16. the center
17. two radii
18. a diameter
19. two chords
20. two central angles
21. the longest chord

Find the measure of each unknown central angle.

22. \[ x \]
\[ \text{measures} \]
\[ 100^\circ \]
\[ 80^\circ \]
\[ 100^\circ \]

23. \[ x \]
\[ \text{measures} \]
\[ 74^\circ \]

24. \[ x \]
\[ \text{measures} \]
\[ 90^\circ \]
\[ 133^\circ \]

25. Draw \( \odot P \) with a diameter \( TV \) and chord \( TM \).
Lesson 4 ~ Circumference and Pi

Find the circumference of each circle. Use 3.14 for \( \pi \).

26. \[ \text{radius} = 8 \text{ cm} \]
27. \[ \text{diameter} = 12 \text{ ft} \]

28. The radius of a bicycle tire is 25.4 cm. What is the circumference of the tire?

29. The first Ferris wheel had a diameter of 250 feet. What was the circumference of the original Ferris wheel?

Find each missing measure. Use 3.14 for \( \pi \).

30. Rudy ran laps on a circular track. He ran 314 meters in one lap. Find the diameter of the track.

31. Samantha made a circular cake for her brother. The circumference of the cake is 50.24 inches. Find the approximate radius of the cake.

32. \( C = 75.36 \text{ cm} \)
   \[ d = ? \]
33. \( C = 25.12 \text{ yd} \)
   \[ r = ? \]
34. \( C = 329.7 \text{ in} \)
   \[ r = ? \]

Lesson 5 ~ Area of a Circle

Find the area of each circle. Use 3.14 for \( \pi \).

35. a circle with a radius of 3 centimeters
36. a circle with a diameter of 24 mm

37. \[ \text{radius} = 6.5 \text{ m} \]
38. \[ \text{radius} = 14 \text{ cm} \]

39. The diameter of a circle is 80 feet. Find the area of the circle.

40. A flying disk has a radius of 6 inches. Find the area of the disk.
Calculate the exact area of each circle.

41. \( \text{Area} = ? \)

42. \( \text{Area} = ? \)

Lesson 6 ~ More Pi

Identify the most appropriate form of π to use in each situation. Explain your choice. Find each missing measure.

43. \( \text{Circumference} = ? \)

44. \( \text{Area} = ? \)

45. \( \text{Area} = ? \)

46. A fish bowl has a circular base with a radius of 14 inches. What is a rough estimate for the area of the base of the fish bowl?

47. Terry wants the most accurate answer possible when finding the area of her circular quilt. It has a radius of 4.7 feet.
   a. Should she use 3.14, \( \frac{22}{7} \), or the \( \pi \) button on her calculator?
   b. Find the area of the quilt using your answer from part a. Round the answer to the nearest hundredth.

48. Give an example of a real-world situation where it would be appropriate to use 3 as an estimate of \( \pi \).

49. Give an example of a real-world situation when an exact answer may be needed to find the area or circumference of a circle.
Lesson 7 ~ Composite Figures

Calculate the area of each shaded region.

50.

51.

52.

53.

54.

55.

Lesson 8 ~ Circle Similarity

Write a ratio comparing the radii of each pair of circles. Write the ratio in simplest terms.

56.

57.

58. Grace has two flower pots. The base of one has a circumference of 16 inches. The other pot’s base has a circumference of 10 inches. Write a ratio comparing the pots’ diameters.

Write a ratio comparing the areas of each pair of circles. Write the ratio in simplest form.

59.

60.
61. ⌀B has a circumference of 72 meters. ⌀X has a circumference of 18 meters. Write a ratio comparing the areas of the circles.

Use a proportion to find each missing measure. As needed, round to the nearest hundredth.

62. \( C = 21.98 \text{ in} \)  \( C = ? \)  \( C = 46 \text{ cm} \)  \( C = 34.5 \text{ cm} \)

64. The radii of two circles have a ratio of 4 : 5. The larger circle has an area of 1000 square centimeters.
   a. What is the ratio of the areas of the circles?
   b. Find the area of the smaller circle.

65. The radii of two circular gears have a ratio of 2 : 7. The area of the small gear is 8 square inches. Find the area of the large gear.

Lesson 9 ~ Area of Sectors

Use a proportion to find the area of each shaded sector. Round answers to the nearest hundredth, if necessary.

66. \( A = 58 \text{ cm}^2 \)
67. \( A = 254.34 \text{ in}^2 \)

68. A circle has an area of 278.9 square meters. Find the area of a sector with a central angle of 66\(^\circ\).

69. A pumpkin pie has an area of 113.04 square inches. The pie is cut into 8 equal slices. Find the area of one slice.

70. The exact area of a circle is \( 78\pi \) square miles. Find the exact area of a sector with a central angle of 120\(^\circ\).

71. A circular waffle has an area of 200.96 square centimeters. Find the area of a slice that has a central angle of 90\(^\circ\).

72. The radius of a circle is 5 inches. Find the area of a sector of the circle with a central angle of 200\(^\circ\). Use 3.14 for \( \pi \).
Manish
Safety and Health Professional
Tillamook, Oregon

I am an occupational safety and health professional. I ensure that my company is meeting Oregon OSHA codes, conducting safety trainings and completing inspections. If an accident happens, I investigate it and handle any workers’ compensation claims. Most importantly, I make sure the workplace is a safe and healthy place for everybody who works there.

I use math to help communicate injury prevention to managers and employees of all levels. For example, I calculate incident rates (the number of injuries or illnesses per 100 employees). I also determine injury causes and calculate percentages by type of injury and body part. Sometimes I analyze chemical and noise exposure levels to make sure that people are not exposed to harmful contaminants. Math helps me know that the workplace is as safe as possible.

Many safety professionals are required to get a specialized degree or certification. I have a Bachelors of Science in Occupational Safety and Health degree from Oregon State University. Some companies require special certifications that can be obtained by attending extra training classes or passing certain exams.

The median salary of an occupational health and safety specialist was $54,920 per year in May 2006. The middle 50 percent earned between $41,800 and $70,230 per year. The lowest 10 percent earned less than $32,230 per year and the highest 10 percent earned more than $83,720 per year. Health and safety specialists can work for private industries, hospitals or governments. Salaries vary depending on which type of employer one works for.

I like being a safety and health professional because I can use math, psychology and business. I am constantly balancing the three subjects and really enjoy the diversity. I’m also proud to know that I am doing my part in keeping Oregonians safe and healthy at work.