BLOCK 2 ~ LINEAR EQUATIONS

SEQUENCES AND SLOPE

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Word Wall

- RATE OF CHANGE
- SLOPE
- y-INTERCEPT
- LINEAR EQUATION
- RECURSIVE SEQUENCE
- RECURSIVE ROUTINE
- SLOPE TRIANGLE
- FUNCTION
### Slope Methods
Create a flip book explaining how to find slope from tables, graphs and ordered pairs.

See page 83 for details.

### Crossing Paths
Determine where two recursive sequences cross paths. Illustrate solutions with tables and graphs.

See page 49 for details.

### Card Game
Make a card game where players create recursive routines and score points.

See page 54 for details.

### Geometric Sequences
Examine recursive sequences involving repeated multiplication. Write equations for these recursive routines.

See page 70 for details.

### Challenging Tables
Find the rates of change and start values in challenging input-output tables.

See page 64 for details.

### Writing Equations from Tables
Create a worksheet to help another student through the process of writing an equation for an input-output table.

See page 74 for details.

### Rate Applications
Find the rates of change in real-world situations. Write application problems.

See page 64 for details.

### Children’s Story
Write a children’s story about recursive sequences. The main character encounters a recursive sequence and develops its equation.

See page 79 for details.

### Similar Slope Triangles
Make discoveries about different size slope triangles formed on the same line.

See page 89 for details.
A recursive sequence is an ordered list of numbers that begins with a start value. Each term in the sequence is generated by applying an operation to the term before it. This same operation is repeated to the resulting value. This process continues to make a sequence of terms.

A recursive routine is described by stating the start value and the operation that is performed to get to the next term. In this case, the recursive routine for the sequence above is:

**Start Value**: 4

**Operation**: Add 3

For each of the following recursive sequences, state the start value, operation and the next three terms.

**a.** 8, 17, 26, 35, 44, …

**b.** 6, 2, −2, −6, −10, …

**Solutions**

**a.**
- Start Value = 8
- Operation = Add 9
- Next three terms: 53, 62, 71

**b.**
- Start Value = 6
- Operation = Subtract 4
- Next three terms: −14, −18, −22

**Example 1**

For each of the following recursive sequences, state the start value, operation and the next three terms.

**a.** 8, 17, 26, 35, 44, …

**b.** 6, 2, −2, −6, −10, …

**Explore!**

**Step 1:** Resting Metabolic Rate (RMR) represents the number of calories your body burns daily when at rest. In the table below, choose the weight and gender that best describes you to determine your approximate RMR. Record your value on your paper.

**Male**

<table>
<thead>
<tr>
<th>Weight (lbs)</th>
<th>RMR (kcal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>1290</td>
</tr>
<tr>
<td>90</td>
<td>1340</td>
</tr>
<tr>
<td>100</td>
<td>1400</td>
</tr>
<tr>
<td>120</td>
<td>1490</td>
</tr>
<tr>
<td>140</td>
<td>1600</td>
</tr>
<tr>
<td>160</td>
<td>1720</td>
</tr>
<tr>
<td>180</td>
<td>1830</td>
</tr>
</tbody>
</table>

**Female**

<table>
<thead>
<tr>
<th>Weight (lbs)</th>
<th>RMR (kcal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>1130</td>
</tr>
<tr>
<td>90</td>
<td>1170</td>
</tr>
<tr>
<td>100</td>
<td>1230</td>
</tr>
<tr>
<td>120</td>
<td>1320</td>
</tr>
<tr>
<td>140</td>
<td>1430</td>
</tr>
<tr>
<td>160</td>
<td>1550</td>
</tr>
<tr>
<td>180</td>
<td>1660</td>
</tr>
</tbody>
</table>
Step 2: Choose an activity that you would most like to participate in from the list below. Record your choice and the calories burned per minute.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Aerobics</th>
<th>Downhill Skiing</th>
<th>Bowling</th>
<th>Horseback Riding</th>
<th>Flag Football</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calories Burned per Minute</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

Step 3: Copy the table shown at right. Insert the name of your activity at the top of the first column.

Step 4: How many calories has your body burned during a full day before you participate in your chosen activity? What is this value called? Where would this fit in the table?

Step 5: Determine the total daily calories burned through the first five minutes of your activity. Continue your calculations to determine the total daily calories burned for 10 minutes, 20 minutes and 30 minutes.

Step 6: If you want to burn 2,000 total calories during one day. How many minutes will you need to participate in your activity? Is this reasonable?

Step 7: Describe the recursive routine for your table (when going up one minute at a time) by giving the start value and the operation that must be performed to arrive at the next term.

Recursion Routine
Start Value: _____
Operation: ______

Example 2
Find the missing values in each sequence. Identify the start value and the operation that must be performed to arrive at the next term.

a. 25, 19, 13, _____, 1, _____, _____
   b. 32, 45, _____, _____, 84, _____

Solutions
a. The numbers in the list are going DOWN 6 each time.
   Start Value: 25
   Operation: Subtract 6
   Completed List: 25, 19, 13, 7, 1, −5, −11

b. The numbers in the list are increasing by 13 each time.
   Start Value: 32
   Operation: Add 13
   Completed List: 32, 45, 58, 71, 84, 97
USING YOUR CALCULATOR... TO CREATE A RECURSIVE ROUTINE

Most scientific and graphing calculators can perform a repeated calculation to create a sequence of numbers.
- Enter the start value.
- Press ENTER or =.
- Enter the operation.
- Press ENTER or = repeatedly to generate the recursive sequence.

EXAMPLE 3

For each sequence, describe the recursive routine by giving the starting value and operation. Give the 15th term.

a. 35, 50, 65, 80, ...
   b. 10, 1, −8, −17, ...

SOLUTIONS

a. Start Value: 35
   Operation: Add 15 (or +15)
   15th Term: 245

b. Start Value: 10
   Operation: Subtract 9 (or −9)
   15th Term: −116

EXERCISES

Copy each sequence of numbers and fill in the missing values. Identify the start value and the operation that must be performed to arrive at the next term.

1. 3, 5, ____ , 9, ____ , ____
2. 125, ____ , 175, 200, ____ , ____

3. 27, 22, 17, ____ , ____ , ____
4. ____ , ____ , −5, −9, −13, ____

5. 23, ____ , 45, 56, ____ , ____
6. 4\(\frac{1}{2}\) , 5, ____ , ____ , 6\(\frac{1}{2}\) , ____

7. ____ , 2.7, 3.1, ____ , 3.9, ____
8. −42, −22, ____ , ____ , ____ , 58
9. Deidre weighs 100 pounds. She enjoys downhill skiing at Mt. Hood Meadows.
   a. Copy the table to the right. Use the table in the EXPLORE! to determine Deidre’s means subtraction. resting metabolic rate (RMR). Insert it in the table for 0 minutes skiing.
   b. How many calories will Deidre burn each minute she skis?
   c. Fill in the total calories she will burn in a full day for each added minute she skis.
   d. Deidre ends up skiing for 40 minutes. How many TOTAL calories did she burn on that day if she participated in no other activities?

<table>
<thead>
<tr>
<th>Minutes Spent Downhill Skiing</th>
<th>Total Daily Calories Burned</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

10. Squares, each 1 centimeter by 1 centimeter, are placed next to each other (one at a time) to form a long strip of squares.

   a. What is the perimeter of the first figure using just one square?
   b. What is the perimeter of the figure using two squares? Three squares?
   c. Draw the next two figures in the pattern. What are the perimeters of these figures?
   d. Write the recursive routine (start value and operation) that describes the perimeters.
   e. Predict the perimeter of the figure that has 14 of these squares in one long strip.

For each sequence describe the recursive routine (start value and operation). Give the 9th term in the sequence.

11. 8, 16, 24, 32 …
12. 10.5, 9.4, 8.3, 7.2 …
13. 9, 5, 1, −3 …
14. \( \frac{1}{3} \), 1, \( \frac{2}{3} \), 2 \( \frac{1}{3} \) …

15. Each block is 1 centimeter by 1 centimeter.
   a. Draw the next two figures in the following pattern.
   b. What is the perimeter of the first figure using just one square?
   c. What is the perimeter of the second figure? The third figure?
   d. Write the recursive routine (start value and operation) that describes the perimeters.
   e. Predict the perimeter of the seventh figure in this pattern.

16. Mary grew strawberries this summer and plans to open a fruit stand to sell her berries. On the first day, she will charge $4.50 for a pound of strawberries. Each day that the fruit stand is open, the price of the berries will go down $0.30.
   a. Write a recursive routine that describes the cost of the strawberries.
   b. Write the sequence of numbers that shows the price of strawberries each day for the first five days the fruit stand is open.
   c. On what day will the price of the strawberries drop below $2.00?
   d. When will she be giving away the strawberries for free?

17. Generate your own recursive sequence. Describe the recursive routine by giving the start value and operation. List the first five numbers in the sequence. What is the 20th term in your sequence?
Copy each table. Complete each table by evaluating the given expression for the values listed.

18. | \( x \) | \( 6x + 7 \) | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

19. | \( x \) | \( 3(x - 1) \) | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

20. Draw a coordinate plane that goes from -10 to 10 on the \( x \)-axis and -10 to 10 on the \( y \)-axis. Graph and label the following points: A(1, 4), B(3, -5), C(0, 2), D(-2, -1), E(-5, 0).

Tic-Tac-Toe ~ Crossing Paths

Recursive routines that represent linear relationships often cross paths or intersect at one point. To find where the sequences intersect, you can create an input-output table or make a graph. Each set of recursive routines below will intersect at one point. Find the point of intersection.

Use an input-output table to determine where the two recursive routines intersect.

1. **Routine A**
   - Start Value = 7
   - Operation = +2

   **Routine B**
   - Start Value = 19
   - Operation = -4

2. **Routine A**
   - Start Value = -9
   - Operation = +7

   **Routine B**
   - Start Value = 19
   - Operation = +3

3. **Routine A**
   - Start Value = 120
   - Operation = -12

   **Routine B**
   - Start Value = -80
   - Operation = +28

Use a graph to determine where the two recursive routines intersect.

4. **Routine A**
   - Start Value = 4
   - Operation = +3

   **Routine B**
   - Start Value = 20
   - Operation = -1

5. **Routine A**
   - Start Value = -2
   - Operation = +5

   **Routine B**
   - Start Value = 13
   - Operation = +2

6. Which method did you prefer for finding the point of intersection? Why?

See Lesson 9 for information on input-output tables and graphing.
Aroldo attended the State Fair with his friends in August. The entry fee was $8 and each ride he went on cost an additional $2. The graph below shows the total Aroldo may have spent depending on the number of rides he went on.

This situation can also be shown using a table. Take each ordered pair on the graph and put it in the corresponding spot in the table.

<table>
<thead>
<tr>
<th>Number of Rides</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost ($)</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>24</td>
<td>26</td>
<td>28</td>
</tr>
</tbody>
</table>

The cost (y-coordinate) based on the number of rides can be described by a recursive routine:

- Start Value: $8
- Operation: Add $2

Notice that the points on the graph form a straight line. The graph shows that there is a linear relationship between the number of rides he went on and his total cost. Aroldo’s total cost at the fair is directly related to the number of rides he went on. When real-life situations are examined mathematically you will find that many can be described as having a linear relationship. Can you think of any other situations that might have a starting value and then go up or down in equal steps?
In a linear relationship, the \( y \)-coordinates follow a recursive routine when the \( x \)-coordinates in a table or scatter plot increase in equal increments. When a graph or table shows the \( x \)-coordinate increasing by 1, the recursive sequence of the \( y \)-coordinates describes the linear relationship.

**Example 1**

Describe the linear relationship given by the \( y \)-coordinates on each graph by stating the recursive routine and the first 10 numbers in the recursive sequence.

**Solutions**

a. The \( y \)-coordinates that are shown are 1, 4, 7, 10, 13 …
   The recursive routine can be described by the following rules:
   - Start Value: 1
   - Operation: Add 3
   Using this recursive routine you can determine that the first 10 \( y \)-coordinates in this sequence are: 1, 4, 7, 10, 13, 16, 19, 22, 25, 28.

b. The \( y \)-coordinates that are shown are 7, 2, –3, –8 …
   The recursive routine can be described by the following rules:
   - Start Value: 7
   - Operation: Subtract 5
   Using this recursive routine you can determine that the first 10 \( y \)-coordinates in the sequence are: 7, 2, –3, –8, –13, –18, –23, –28, –33, –38.

Sometimes it is useful to generate a table to represent the ordered pairs of recursive sequences shown on a linear plot. This can be done by creating an input-output table for the \( x \)- and \( y \)-coordinates. For example, the ordered pairs of Part A in Example 1 can be converted to a table by recording each \( x \)-coordinate with its corresponding \( y \)-coordinate.
EXAMPLE 2

Jerome throws the shot put for Newbridge High School. Coming into the season, his personal best was 42 feet. Each week, his shot put throws increase by 0.5 feet. Create a linear plot that represents this situation. Write a recursive routine to describe it.

SOLUTION

Start Value: 42 Feet
Operation: Add 0.5 Feet

Recursive Routines for Linear Relationships

Start Value: corresponds to an x-value of 0.
Operation: the amount the y-value increases or decreases for each unit on the x-axis.

EXERCISES

Describe the linear relationship given by the y-coordinates on each linear plot by stating the start value and operation. Create an input-output table showing the ordered pairs on each linear plot.

1.

2.

3.

4.
5. Kirk chose to go down the steepest, fastest water slide in the aquatic park. The linear plot shows how high off the ground Kirk is, based on the number of seconds he has been on the slide.

a. Copy the input-output table shown and fill in all the ordered pairs shown on the linear plot.

<table>
<thead>
<tr>
<th>Time (seconds), $x$</th>
<th>Feet off the Ground, $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

b. Write the recursive routine for Kirk's ride down the slide.

c. What does the start value represent in real life?

d. How long does it take for Kirk to get to the bottom of the slide?

Create a linear plot for the first five ordered pairs for the given recursive routine. Remember that the start value corresponds to an $x$-value of 0.

10. Start Value: 8
    Operation: Subtract 3

11. Start Value: 1
    Operation: Add $\frac{1}{2}$

12. Create a recursive routine by picking a start value and operation (adding or subtracting).
    a. Record your recursive routine on your homework.
    b. Create a linear plot for at least five points in your recursive sequence.
    c. Create an input-output table that shows the $x$- and $y$-coordinates for the linear plot you generated.
Write and simplify an expression for the area of each figure.

Area of Rectangle = $bh$  \hspace{1cm} Area of Triangle = $\frac{1}{2}bh$

13. \hspace{1cm} 14.
\[2x - 3 \hspace{1cm} x + 7\]
\[6 \hspace{1cm} 10\]

15. \hspace{1cm} 16.
\[-3x + 26 \hspace{1cm} 40\]
\[x + 10 \hspace{1cm} \]

**Tic-Tac-Toe ~ Card Game**

Use a regular deck of playing cards for this activity. Take out all the face cards (Jacks, Queens and Kings). Create a card game that can be played with two people. The card game must make the players use recursive routines to earn "points". Be creative with your rules.

Some ideas to think about:

- How many cards does each person start with?
- Do some colors and/or suits represent negative integers or subtraction?
- How will the cards be used to determine a start value of a recursive routine?
- How will the cards be used to determine an operation of a recursive routine?
- How are points scored or how does the person progress towards a finish line?
- Do players ever have to find a specific term in the sequence based on a number they draw from the deck?

Once you have designed your game, ask two different pairs of people to try it out. Ask each player to write a short review of your game once they have played it. Read the reviews and write a one-page paper summarizing the feedback. Also include in your paper any changes you would make in the rules before the game was played again. Turn in your paper along with the original set of rules for your game.
Recursive routines are useful when dealing with a variety of real-world situations. Recursive routines can be illustrated with graphs, tables and by words. Using multiple ways of showing a recursive routine helps to reach a variety of audiences. It is important to think about what type of graphic (table, graph, words, etc.) best illustrates each situation.

William and his sister, Jennifer, each worked summer jobs. William mowed lawns in his neighborhood. Jennifer baby-sat for two different families. By the end of the summer, William had put $410 in a savings account. Jennifer put $275 in her own account. After school started, Jennifer continued baby-sitting and earned $20 per week. She put all of her earnings in her savings account. William stopped working and withdrew $15 per week from his savings account for spending money.

**Step 1:** Write a recursive routine (start value and operation) for the amount in William’s savings account each week after school begins.

**Step 2:** Write a recursive routine for the amount in Jennifer’s savings account each week after school begins.

**Step 3:** Copy the input-output tables shown below and fill in each for the first 10 weeks after school starts.

<table>
<thead>
<tr>
<th>Weeks After School Starts</th>
<th>William’s Total Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Weeks After School Starts</th>
<th>Jennifer’s Total Savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

**Step 4:** On the SAME first-quadrant coordinate plane, graph William and Jennifer’s total savings for the first ten weeks. Use ◆ to designate Jennifer’s amounts and ■ to represent William’s amounts. Put weeks on the x-axis and $$ on the y-axis.

**Step 5:** After what week does Jennifer have more money than her brother? Which illustration (table, graph or recursive routine) best shows this?
EXAMPLE 1

Matt pays a fee of $25 per month for his cell phone plan. He is charged $0.15 per text message he sends or receives.

a. Write a recursive routine that describes Matt’s monthly cell phone bill based on the number of text messages he sent or received.

b. Create an input-output table for the first ten text messages.

c. Create a linear plot that shows his total monthly bill for up to ten text messages.

d. Determine Matt’s total bill for the month of January if he sent or received 42 text messages.

SOLUTIONS

a. Start Value = $25
    Operation = Add $0.15 (or + 0.15)

<table>
<thead>
<tr>
<th>Text Messages Sent or Received</th>
<th>Total Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$25</td>
</tr>
<tr>
<td>1</td>
<td>$25.15</td>
</tr>
<tr>
<td>2</td>
<td>$25.30</td>
</tr>
<tr>
<td>3</td>
<td>$25.45</td>
</tr>
<tr>
<td>4</td>
<td>$25.60</td>
</tr>
<tr>
<td>5</td>
<td>$25.75</td>
</tr>
<tr>
<td>6</td>
<td>$25.90</td>
</tr>
<tr>
<td>7</td>
<td>$26.05</td>
</tr>
<tr>
<td>8</td>
<td>$26.20</td>
</tr>
<tr>
<td>9</td>
<td>$26.35</td>
</tr>
<tr>
<td>10</td>
<td>$26.50</td>
</tr>
</tbody>
</table>

d. Use a calculator. Enter the start value, 25, and press ENTER or =. Then enter your operation, + 0.15, and press ENTER or = 42 times. You should arrive at the answer of $31.30.

A few things to remember when creating tables and graphing:

- Always put the ‘counter’ in the first column of a table and on the x-axis of a graph. The ‘counter’ is the item the y-value is dependent on. It will start at zero and go up by one each time.

- Most real-world situations take place in the first quadrant. Think about your situation before graphing and decide if negative numbers would ever make sense. For example, you will not have a negative amount for the cost of a cell phone bill, so you will only need to use the first quadrant.

- Choose a range (lowest to the highest number) for the y-axis that allows the viewer of your graph to see all points easily. Also, make sure your increments on the y-axis are reasonable.
Determine an appropriate range for the $y$-axis. State what increments you would use on the graph.

1. **Minutes** | **Distance Traveled**
--- | ---
0 | 8
1 | 20
2 | 32
3 | 44
4 | 56
5 | 68

2. **Sales Made** | **Salary**
--- | ---
0 | $120
1 | $150
2 | $180
3 | $210
4 | $240
5 | $270

3. **Years** | **Car’s Worth**
--- | ---
0 | $12,000
1 | $10,500
2 | $9,000
3 | $7,500
4 | $6,000
5 | $4,500

4. Jackson got his driving license one year ago. When Jackson got his driver’s license, his car insurance cost $82 per month. Each time he gets a speeding ticket, his insurance goes up $26 per month.
   a. Write a recursive routine that describes Jackson’s monthly car insurance bill based on the number of tickets he has received.
   b. Create an input-output table for 0 to 5 speeding tickets.
   c. Create a linear plot that shows his total monthly bill through the first five tickets.
   d. Use a calculator to determine Jackson’s total monthly bill in the first year of driving if he has received 12 speeding tickets.

5. Maggie bought a laptop computer for $799. Each year, the value of her laptop decreases by $70.
   a. Write a recursive routine that describes the value of Maggie’s laptop based on the number of years she has owned it.
   b. Create an input-output table for the value of the laptop for 0 to 5 years.
   c. Create a linear plot that shows the value of the laptop through the first five years.
   d. Use a calculator to determine how many years it will take before the laptop is not worth anything.

6. Fran borrowed $200 from his parents to buy a mountain bike. Each week, he uses $14 of his allowance to pay back his parents.
   a. Write a recursive routine that describes the total amount Fran owes his parents based on the number of weeks that have passed since he borrowed the money.
   b. Create an input-output table that shows the amount he still owes for 0 to 5 weeks.
   c. Create a linear plot that shows the amount Fran still owes his parents through the first five weeks.
   d. Use a calculator to determine how many weeks it will take before Fran has paid back his parents. How much was his last payment?
7. Quincy hiked up a slope in Desert Shores, California (one of the few places below sea level in the United States). He began at an elevation 61 feet below sea level. Each minute that he hiked, he rose 7 feet in elevation.
   a. Write a recursive routine that describes Quincy’s elevation based on the number of minutes he hiked.
   b. Create an input-output table to find his elevation for 0 to 10 minutes of hiking.
   c. Create a linear plot that shows Quincy’s change in elevation through the first 10 minutes.
   d. How many minutes did it take for Quincy to get above sea level?

8. Victor and Mike had a pizza-eating contest. Victor had already eaten three pieces when the competition started. Mike had only eaten one piece. Once the competition started, Victor was able to eat $\frac{1}{2}$ of a piece every minute. Mike was able to eat a little faster. He ate $\frac{3}{4}$ of a piece every minute.
   a. Write two recursive routines, one that describes Victor’s pizza-eating and the other describing Mike’s pizza-eating. Label them accordingly.
   b. The pizza-eating competition lasted for 8 minutes. Create two input-output tables that show the number of pieces each boy had eaten for each of the first 8 minutes.
   c. Who won the competition at the end of 8 minutes?

9. When Kathy was born, her grandparents started an account for her college education with $1,000 in it. Each year, on her birthday, they add $250.
   a. Write a recursive routine that gives the amount of money in Kathy’s account based on her age, not including interest.
   b. Determine the total amount her grandparents will have contributed after her 18th birthday.
   c. Overall, the entire account earned 28% interest. Determine the total amount the account was worth when she withdrew it after her eighteenth birthday.

**REVIEW**

Find the missing values in each sequence. Identify the start value and the operation that must be performed to arrive at the next term.

10. $-14, -11, _____, _____, -2, _____
11. $5.8, 4.6, _____, 2.2, _____, _____
12. $9, _____, 21, _____, 33, _____
13. $\frac{1}{3}, 1, ___, 2\frac{1}{3}, ___, ___$

Solve each equation. Check the solution.

14. $x + 28 = 102$
15. $\frac{x}{6} = -7$
16. $-3x + 5 = 38$
17. $5x + 7 = 7x - 9$
18. $3 = \frac{x}{2} - 1.5$
19. $2x + 7 = 4$
So far in Block 2, you have been looking at many different recursive routines. Each recursive sequence you have examined represents a linear relationship. This means that when you plot the points of the sequence on a coordinate plane, the points fall into a straight line. For each recursive routine, you have been able to define the operation that allows you to move from one number in the sequence to the next because you have been told how much to increase or decrease for each step.

There are many situations where the operation is not given for just one step. For example:

Colton eats 560 calories in 10 minutes.
Luke paints 72 pictures in 3 days.
Karen goes down 90 steps in 4 minutes.

In order to determine the operation needed for each situation, you must calculate the rate of change. The rate of change can be found by calculating the change in the output (or y-values) divided by the change in input (x-values). It is also called the unit rate. It is important to think about which term is being used as the “counter” and place that term in the denominator of the rate because it is your x-value. Time is the most common ‘counter’.

You must also determine if a situation is giving you increasing numbers in the recursive sequence (pictures painted per day, calories eaten per minute) or decreasing numbers in the sequence (steps descended per minute). This will help in deciding if you are adding or subtracting your “rate of change” amount.

<table>
<thead>
<tr>
<th>Situation</th>
<th>Rate</th>
<th>Rate of Change</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colton eats 560 calories in 10 minutes.</td>
<td>560 calories</td>
<td>56 calories</td>
<td>Add 56</td>
</tr>
<tr>
<td></td>
<td>10 minutes</td>
<td>1 minute</td>
<td></td>
</tr>
<tr>
<td>Luke paints 72 pictures in 3 days.</td>
<td>72 pictures</td>
<td>24 pictures</td>
<td>Add 24</td>
</tr>
<tr>
<td></td>
<td>3 days</td>
<td>1 day</td>
<td></td>
</tr>
<tr>
<td>Karen goes down 90 steps in 4 minutes.</td>
<td>90 steps</td>
<td>−22.5 steps</td>
<td>Subtract 22.5</td>
</tr>
<tr>
<td></td>
<td>4 minutes</td>
<td>1 minute</td>
<td></td>
</tr>
</tbody>
</table>

**Increasing?**

**Decreasing?**
Lesson 11 ~ Rate Of Change

Determine the rate of change for each situation. State the operation that would occur in the recursive routine.

a. Jessica loses $580 in 20 days in the stock market.
b. Patrick earned $53 for delivering 10 packages.

**Solutions**

a. The ‘counter’ is the number of days so this term goes in the denominator.

\[ \frac{-580}{20 \text{ days}} = \frac{-29}{1 \text{ day}} \]

She is losing money, so the operation involves subtraction.

Operation = Subtract -29

b. The ‘counter’ is the number of packages.

\[ \frac{53}{10 \text{ packages}} = \frac{5.30}{1 \text{ package}} \]

He is earning money, so the operation involves addition.

Operation = Add 5.30

In some situations, information will be given to you in an input-output table. In those cases, you must be able to determine the rate of change by locating numbers on the table that will lead you to the rate of change.

**Example 1**

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>13</td>
</tr>
</tbody>
</table>

**Change in x-values = +1**

**Change in y-values = +3**

Calculate the rate of change.

\[ \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{+3}{+1} = +3 \]

The start value is 4 because it is the y-value that is paired with an x-value of 0.
In some tables the $x$-coordinate of 0 is not listed. This means that the start value is not given. In order to find the start value you must first find the rate of change. Use the rate of change to work forward or backward to find the $y$-value that is paired with 0.

**EXAMPLE 3**  
The rate of change in the table is $-2$. Find the start value.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

**Solution**

Rewrite the table to include $x$-coordinates to 0.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

The rate of change is $-2$. Work backwards to get to the $x$-coordinate of 0 by doing the opposite of the rate of change. Add 2 for each step.

The start value is 10.

**EXAMPLE 4**  
Find the start value and rate of change for the input-output table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$-9$</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>9</td>
<td>35</td>
</tr>
<tr>
<td>12</td>
<td>47</td>
</tr>
</tbody>
</table>

**Solution**

Find the rate of change by selecting two pairs of numbers. Find the change in $x$ and the change in $y$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$-9$</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>27</td>
</tr>
<tr>
<td>9</td>
<td>35</td>
</tr>
<tr>
<td>12</td>
<td>47</td>
</tr>
</tbody>
</table>

Change in $x$-values $= +2$  
Change in $y$-values $= +8$

Calculate the rate of change.

\[
\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}} = \frac{+8}{+2} = +4
\]
Use the rate of change to work forwards from $x = -2$ to find the $y$-value paired with the $x$-coordinate of 0.

The start value is $-1$.
The rate of change is $+4$.

**EXERCISES**

Determine the rate of change for each situation.

1. George collected 18 bugs in 9 days.

2. Over 6 days Theo spent $336.

3. Michiko took 760 steps during a 15 minute run.


Determine the rate of change and start value for each table.

5. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
</tr>
</tbody>
</table>

6. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>12</td>
</tr>
<tr>
<td>-1</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

7. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

8. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>16</td>
</tr>
</tbody>
</table>

9. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
</tr>
<tr>
<td>4</td>
<td>-9</td>
</tr>
<tr>
<td>6</td>
<td>-13</td>
</tr>
<tr>
<td>9</td>
<td>-19</td>
</tr>
</tbody>
</table>

10. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>3</td>
</tr>
<tr>
<td>-1</td>
<td>3.5</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

Use the given rate of change and start value to complete each table.

11. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Rate of Change = +8
Start Value = 1

12. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>4.8</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>12.6</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

Rate of Change = +2.6
Start Value = 4.8

13. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>18</td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Rate of Change = −5
Start Value = 8
14. Jim-Bob’s Car Rental Company charges a set fee for renting a car and an additional amount per mile driven. Frank has rented from Jim-Bob’s three times and his charges are shown in the table to the right.
   a. How much does Jim-Bob charge per mile driven?
   b. What is the set fee for renting a car at Jim-Bob’s?
   c. How much would a car rental cost if Frank drove 30 miles?

<table>
<thead>
<tr>
<th>Miles Driven</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$17.60</td>
</tr>
<tr>
<td>10</td>
<td>$20.00</td>
</tr>
<tr>
<td>22</td>
<td>$24.80</td>
</tr>
</tbody>
</table>

15. Mark moved into a new house and believes his bedroom will soon be taken over by ants. In the table shown below, Mark records the number of ants in his bedroom on different days since he moved in.

<table>
<thead>
<tr>
<th>Days Since Mark Moved In</th>
<th>Number of Ants</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>66</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
</tr>
<tr>
<td>9</td>
<td>138</td>
</tr>
<tr>
<td>13</td>
<td>186</td>
</tr>
<tr>
<td>15</td>
<td>210</td>
</tr>
</tbody>
</table>

   a. How many ants are moving into Mark’s bedroom each day?
   b. How many ants were in his room when he first moved in?
   c. If this pattern continues, how many ants will be in his room after 3 weeks?

16. Mario rides his scooter to work each day. He is able to travel 0.5 miles per minute. He lives 4.7 miles from work.
   a. Copy the table and fill in the distance Mario has left to work based on each minute he has traveled from his home. Continue the table until he has arrived at work.
   b. What is the rate of change in this situation?
   c. Can you figure out exactly (to the second) how long Mario’s trip is? If so, how long is it?

<table>
<thead>
<tr>
<th>Minutes Traveled</th>
<th>Distance to Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.7</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

**REVIEW**

Each input-output table represents a real-world situation. Determine an appropriate range for the y-axis. State what increments you would use on the y-axis.

17. | Hours | Distance Traveled |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>1</td>
<td>75</td>
</tr>
<tr>
<td>2</td>
<td>105</td>
</tr>
<tr>
<td>3</td>
<td>135</td>
</tr>
<tr>
<td>4</td>
<td>165</td>
</tr>
<tr>
<td>5</td>
<td>195</td>
</tr>
</tbody>
</table>

18. | Days | Plant Height |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>0.6</td>
</tr>
<tr>
<td>4</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>1.0</td>
</tr>
</tbody>
</table>

19. | Lawns Mowed | Profit  |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−$50</td>
</tr>
<tr>
<td>1</td>
<td>−$30</td>
</tr>
<tr>
<td>2</td>
<td>−$10</td>
</tr>
<tr>
<td>3</td>
<td>$10</td>
</tr>
<tr>
<td>4</td>
<td>$30</td>
</tr>
<tr>
<td>5</td>
<td>$50</td>
</tr>
</tbody>
</table>
Tic-Tac-Toe ~ Challenging Tables

Find the equation that represents the linear relationship in each table. Show all work.

1. \[
\begin{array}{c|c}
 x & y \\
-3 & 1.5 \\
0 & 2.7 \\
2 & 3.5 \\
6 & 5.1 \\
8 & 5.9 \\
\end{array}
\]

2. \[
\begin{array}{c|c}
 x & y \\
-8 & 1500 \\
-3 & 875 \\
4 & 0 \\
9 & -625 \\
14 & -1250 \\
\end{array}
\]

3. \[
\begin{array}{c|c}
 x & y \\
-15 & 0 \\
-12 & 1 \\
-10 & 1\frac{1}{3} \\
-6 & 3 \\
-1 & 4\frac{2}{3} \\
\end{array}
\]

4. \[
\begin{array}{c|c}
 x & y \\
-8 & 84 \\
2 & 44 \\
7 & 24 \\
10 & 12 \\
15 & -8 \\
\end{array}
\]

5. \[
\begin{array}{c|c}
 x & y \\
1 & 0.3 \\
3 & 0.06 \\
5 & -0.18 \\
7 & -0.42 \\
11 & -0.9 \\
\end{array}
\]

6. \[
\begin{array}{c|c}
 x & y \\
-2 & 2 \\
3 & 4\frac{1}{2} \\
7 & 6\frac{1}{2} \\
10 & 8 \\
12 & 9 \\
\end{array}
\]

Tic-Tac-Toe ~ Rate Applications

Rates of change are calculated in many situations. Determine the rate of change in each situation given below. Assume each situation forms a linear relationship.

1. Ryan and Silas each bought a package of paper. Ryan bought 7 pencils with his paper for $1.96. Silas bought 12 of the same pencils with his paper for $3.36. Find the cost per pencil.

2. Kendra was at an elevation of −45 feet after 5 minutes of hiking. She was at −3 feet after eleven minutes. What was her rate of change in elevation in feet per minute?

3. Owen was 5.2 miles from home 10 minutes after school was over. He arrived home 30 minutes after school was over. What was his rate of speed going home from school?

Create a worksheet of 10 of your own rate problems. Type or neatly print the problems. Include the answers on a separate sheet of paper.
In this lesson you will learn how to take a recursive routine and determine the linear equation that represents the situation. When the solutions of a linear equation are graphed, they form a line. Almost all linear equations are also linear functions. A function is a pairing of input and output values according to a specific rule.

One common form of a linear equation is $y = b + mx$ where $b$ represents the start value and $m$ represents the rate of change.

**EXPLORE!**

Vendors at the local Farmer’s Market sell a variety of produce and homemade products. Examine each vendor’s situation and write a linear equation that models their profits.

**Step 1:** Peter sells corn at his booth in the Farmer’s Market. His start-up cost for his business was $200 which he spent on seeds, fertilizer and water. What number would represent the start value for his situation: 200 or −200? Why?

**Step 2:** Peter earns $0.25 for each ear of corn he sells. What number represents his rate of change: 0.25 or −0.25? Why?

**Step 3:** One form of a linear equation is $y = b + mx$. The $b$ represents the start value and $m$ represents the rate of change. Write a linear equation to represent Peter’s total profits.

**Step 4:** Nakisha sells homemade candles at the Farmer’s Market. She recorded her total profit for the first five candles sold in the table at the right. What is the start value for her business? What is the rate of change?

**Step 5:** Write a linear equation in the form of $y = b + mx$ to represent Nakisha’s total profits at the Farmer’s Market.

**Step 6:** Nakisha sold a total of 24 candles. Use your linear equation to determine her total profits.

<table>
<thead>
<tr>
<th>Candles Sold, x</th>
<th>Total Profits, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>−$10</td>
</tr>
<tr>
<td>1</td>
<td>−$7</td>
</tr>
<tr>
<td>2</td>
<td>−$4</td>
</tr>
<tr>
<td>3</td>
<td>−$1</td>
</tr>
<tr>
<td>4</td>
<td>$2</td>
</tr>
<tr>
<td>5</td>
<td>$5</td>
</tr>
</tbody>
</table>
Step 7: Luke opened a booth at the Farmer’s Market. He had a positive balance of $300 in his bank account. He paid $50 each week to rent his booth. He had trouble selling his products. His total savings is shown in the graph at the right. What is the recursive routine for this graph? Write a linear equation to represent this situation.

Step 8: One student summarized writing linear equations with the graphic below. How would you know whether to put a + or − between the start value and the rate of change in different situations?

\[ y = \text{Start Value} \pm \text{Rate of Change} \times x \]

Step 9: Write a linear equation in the form \( y = b + mx \) for each recursive routine:

- a. Start Value = 4  Rate of Change = +5
- b. Start Value = −7 Rate of Change = +2
- c. Start Value = 0  Rate of Change = −12
- d. Start Value = −8 Rate of Change = +0

EXPLORATION

You have found the start value in a recursive routine by looking at tables. It is the number paired with an \( x \)-value of 0. The start value can also be located on a graph as the point on the \( y \)-axis.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td>−5</td>
</tr>
<tr>
<td>0</td>
<td>−2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Linear equations represent the start value using the variable \( b \). The start value (\( b \)) is also called the \( y \)-intercept. The \( y \)-intercept is the value of \( y \) where the graph crosses the \( y \)-axis or the number paired with an \( x \)-value of 0 in a table.
EXAMPLE 1
Write the linear equation for each recursive routine.

a. Rate of Change = +4  
   Start Value = −23

b. Rate of Change = −0.3  
y-intercept = 2.8

c. Rate of Change = +2  
y-intercept = 0

d. Rate of Change = 0  
   Start Value = 7

SOLUTIONS

a. Insert the rate of change and the start value into the equation $y = b + mx$.
   Remember, the rate of change is always the coefficient of the $x$-variable because the $x$-variable counts how many “steps” to take.
   
   $y = −23 + 4x$

b. Insert the rate of change and the $y$-intercept into the equation.
   Remember, the $y$-intercept is just another way to refer to the start value.
   
   $y = 2.8 − 0.3x$

c. Insert the rate of change and the $y$-intercept into the equation.
   
   $y = 0 + 2x \rightarrow y = 2x$

   Adding 0 does not affect the value of the equation. It does not need to be written.

d. A rate of change equal to 0 cancels out the $x$-term.
   
   $y = 7 + 0x \rightarrow y = 7$

EXAMPLE 2
Determine the rate of change and $y$-intercept (start value) for each table. Write a linear equation that represents each table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>−1</td>
<td>−2</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>6</td>
<td>33</td>
</tr>
<tr>
<td>4</td>
<td>−2</td>
<td>9</td>
<td>48</td>
</tr>
<tr>
<td>5</td>
<td>−4</td>
<td>12</td>
<td>63</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>−2</td>
</tr>
<tr>
<td>5</td>
<td>−4</td>
</tr>
</tbody>
</table>

a. The start value is the $y$-value that matches with the $x$-value of 0. The start value is 6.

The rate of change is calculated by determining the change in the $y$-values divided by the change in the $x$-values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>−2</td>
</tr>
<tr>
<td>5</td>
<td>−4</td>
</tr>
</tbody>
</table>

Change in $x$-values = +1  
Change in $y$-values = −2

Rate of Change = \[
\frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-values}} = \frac{-2}{+1} = -2
\]

Linear Equation: $y = 6 − 2x$
**EXAMPLE 2**

**SOLUTIONS (CONTINUED)**

b. Calculate the rate of change first when the start value is not given in the table.

\[
\begin{array}{c|c}
 x & y \\
-1 & -2 \\
1 & 8 \\
4 & 23 \\
6 & 33 \\
9 & 48 \\
12 & 63
\end{array}
\]

Change in \(x\)-values = +2  \hspace{1cm} \text{Change in } y\text{-values} = +10

\[
\text{Rate of Change} = \frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-values}} = \frac{+10}{+2} = +5
\]

To find the start value, use the rate of change to find the \(y\)-value that is paired with the \(x\)-value of 0.

\[
\begin{array}{c|c}
 x & y \\
-1 & -2 \\
0 & 3
\end{array}
\]

+5

A rate of change of +5 means the \(y\)-value increases by 5 each time the \(x\)-value increases by 1. The \(x\)- and \(y\)-values one step before zero are given. Add 5 once to the \(y\)-value to get the start value: \(-2 + 5 = 3\).

Linear Equation: \(y = 3 + 5x\)

---

**EXERCISES**

Write the equation for each recursive routine.

1. Rate of Change = +8  
   Start Value = −6

2. Rate of Change = \(-\frac{1}{2}\)  
   \(y\)-Intercept = \(3\frac{1}{4}\)

3. Rate of Change = +7.1  
   Start Value = 0

4. Rate of Change = −3  
   \(y\)-Intercept = 7

5. Start Value = −10  
   Rate of Change = 0

6. \(y\)-Intercept = 120  
   Rate of Change = −54

Determine the rate of change and \(y\)-intercept for each table. Write a linear equation that represents each table.

7. \[
\begin{array}{c|c}
 x & y \\
0 & 4 \\
1 & 12 \\
2 & 20 \\
3 & 28 \\
4 & 36
\end{array}
\]

8. \[
\begin{array}{c|c}
 x & y \\
0 & 12 \\
1 & 11 \\
2 & 10 \\
3 & 9 \\
4 & 8
\end{array}
\]

9. \[
\begin{array}{c|c}
 x & y \\
-2 & 2 \\
-1 & 4 \\
0 & 6 \\
1 & 8 \\
3 & 12
\end{array}
\]
Determine the rate of change and y-intercept for each table. Write a linear equation that represents each table.

10.  | x | y  |
     | -1| 29.5 |
     | 0 | 31   |
     | 3 | 35.5 |
     | 4 | 37   |
     | 7 | 41.5 |

11.  | x | y  |
     | -2| -1  |
     | 2 | 15  |
     | 5 | 27  |
     | 7 | 35  |
     | 10| 47  |

12.  | x | y  |
     | 4 | 1   |
     | 6 | 2   |
     | 10| 4   |
     | 13| 5.5 |
     | 18| 8   |

13. Kirsten was able to finish 12 of her math homework problems at school. At home, she can do 4 problems every 2 minutes.
   a. How many problems does Kirsten complete each minute?
   b. What is Kirsten’s start value for her homework on this particular day?
   c. Write a linear equation that represents this situation.
   d. What do the x-values represent in this equation?
   e. What do the y-values represent in this equation?

14. Jermaine wants to write a linear equation that will help him calculate how much money he has saved based on the number of days he has been saving. He begins with nothing in his savings. He saves $6 per day.
   a. What is the linear equation that represents this situation?
   b. How much will he have saved after 12 days?

15. Jack left a bottle of water sitting on the counter. When he first measured the temperature, it was 65°F. Each hour, he measured the temperature. It remained at 65°F.
   a. What is the y-intercept in this situation?
   b. What is the rate of change?
   c. Write a linear equation to represent the water’s temperature based on the number of hours that have passed.

16. Shannon climbed to the top of a very tall slide and sent a ball down the slide. The top of the slide is 32 feet off the ground. The ball took only 4 seconds to make it to the bottom of the slide.
   a. What is the y-intercept in this situation?
   b. What is the rate of change in feet per second?
   c. Write a linear equation to represent the ball’s height off the ground based on the number of seconds it has traveled.

Copy each table. Determine the rate of change and y-intercept. Fill in the missing values and write the linear equation that represents the table.

17.  | x | y  |
     | 0 | -1  |
     | 1 | 2   |
     | 2 | 5   |
     | 3 |     |
     | 4 |     |

18.  | x | y  |
     | -1| 24  |
     | 0 | 23.5|
     | 1 | 23  |
     | 2 |     |
     | 3 |     |

19.  | x | y  |
     | 0 | 14  |
     | 1 | 25  |
     | 2 | 36  |
     | 5 |     |
     | 7 |     |
Copy and complete each table by evaluating the given expression for the values listed.

### Table 20.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0.5 ( x - 1 )</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>−1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 21.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -2x + 7 )</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>−1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Determine the rate of change for each situation.

22. $72 in 8 hours
23. 140 in 4 hours
24. 963 words in 9 minutes
25. 50 points for 4 assignments

---

**Tic-Tac-Toe ~ Geometric Sequences**

A geometric sequence is a list of numbers created by multiplying the previous term in the sequence by a common ratio. A geometric sequence can be described by a recursive routine with a start value and an operation. For example:

- **Start Value = 4**
- **Operation = \( \times 2 \)**
- **Sequence:** 4, 8, 16, 32, 64, …

Since geometric sequences are not linear, they will not be represented by the equation \( y = b + mx \). Geometric sequences can be represented by the equation \( y = b \cdot m^x \) where \( b \) is the start value and \( m \) is the amount used for the repeated multiplication. For the example above, an equation to represent this sequence is \( y = 4 \cdot 2^x \).

Copy and complete each geometric sequence. Give the start value and the operation.

1. 5, 15, _____, 135, _____
2. 2, −4, _____, −16, 32, _____
3. 120, _____, 30, 15, _____
4. −100, 10, _____, _____, _____

5. Find the 8th term in each geometric sequence in Exercises #1 through #4.

6. Write an equation to represent each geometric sequence in #1-4.

7. Create two of your own geometric sequences. Record the start value, operation and the tenth term. Write an equation representing each sequence.
Lesson 13 ~ Input - Output Tables From Equations

The Commutative Property of Addition states that numbers can be added in any order. This can be applied in a linear equation. In Lesson 12, linear equations were shown in the form \( y = b + mx \). Based on the Commutative Property, this equation can also be written \( y = mx + b \). For example:

\[
\begin{align*}
y &= 7 + 2x & \rightarrow & & y &= 2x + 7 \\
y &= 4 - 3x & \rightarrow & & y &= -3x + 4
\end{align*}
\]

Notice that the “−” belongs to the rate of change and must move with it.

**EXPLORE!**

Use the linear equations given in the box to complete this activity.

**Step 1:** List the equation(s) that have a negative rate of change.

**Step 2:** List the equations(s) that have a positive \( y \)-intercept.

**Step 3:** Which equation has a start value of zero? How do you know?

**Step 4:** Which equation has a rate of change equal to zero? How do you know?

**Step 5:** Create your own linear equation that fits the given description.

- **a.** A positive start value and negative rate of change.
- **b.** A rate of change equal to zero.
- **c.** A start value equal to zero.
- **d.** A negative rate of change and a negative \( y \)-intercept.

There are times when you are given the equation that describes a relationship between two pieces of information. An equation is useful in creating other ways to display the information. The most common ways of displaying data are through graphs, tables and words.

Over summer break Josie went to the mall with her friends. At noon, they left the mall and began walking. The equation \( y = 2 + 3x \) represents Josie’s distance \( (y) \) from her home. The \( x \) represents the number of hours she has walked.
If Josie walked for at least three hours, an input-output table of values can be created that shows how far Josie is from home based on how long she has been walking. This can be done by identifying the \( y \)-intercept and rate of change of the situation.

\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0 & 2 \\
1 & 5 \\
2 & 8 \\
3 & 11 \\
\hline
\end{array}
\]

Graphs are another way to visually display equations. Take each ordered pair from the table and graph it on a coordinate plane. Since Josie is continually walking, the points can be connected to form a straight line.

**Example 1**

**Use the linear equation to complete the input-output table.** \( y = 3x + 8 \)

**Solution**

Substitute each \( x \)-value into the equation to determine the \( y \)-values.

\[
\begin{array}{|c|c|c|}
\hline
x & 3x + 8 & y \\
\hline
-4 & 3(-4) + 8 & -4 \\
7 & 3(7) + 8 & 29 \\
16 & 3(16) + 8 & 56 \\
29 & 3(29) + 8 & 95 \\
\hline
\end{array}
\]

Any input-output table can be turned into ordered pairs and graphed. When dealing with linear equations, graphing is a great way to double-check the calculations as all points should be in a straight line.
EXERCISES

Determine the rate of change and the \( y \)-intercept from the given equations.

1. \( y = 8 + 2x \)  
2. \( y = 3x - 11 \)  
3. \( y = x - 4 \)  
4. \( y = 5 - 4x \)  
5. \( y = -\frac{1}{4}x \)  
6. \( y = -1 \)  
7. \( y = \frac{2}{3}x - 8 \)  
8. \( y = 6 \)  
9. \( y = 2 - \frac{4}{7}x \)

Given the equation, copy and complete the input-output tables.

10. \( y = 2x - 3 \)  
11. \( y = x + 9 \)  
12. \( y = -10 + 6x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( 2x - 3 )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x + 9 )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-4</td>
<td>-4</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>21</td>
<td>21</td>
<td>21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -10 + 6x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>-7</td>
<td>-7</td>
</tr>
<tr>
<td>-3</td>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

13. \( y = -3x \)  
14. \( y = \frac{1}{2}x + 1 \)  
15. \( y = 5 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-6</td>
</tr>
<tr>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-4</td>
</tr>
<tr>
<td>-3</td>
<td>-3</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

16. Gracie planted a marigold in June. She measured its height each week and found that the height of the plant could be represented by the equation \( y = 3 + 0.5x \) where \( x \) represents the number of weeks that have passed and \( y \) represents the height of the plant in inches.

   a. Copy and complete the table to show the height of the marigold through the summer.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

b. Graph the ordered pairs on a coordinate plane.
17. Star is able to run 6.8 meters per second when she is sprinting. She wants to figure out how many meters \((y)\) she can run based on the number of seconds \((x)\) she has run. She developed an equation to help her: \(y = 6.8x\).

a. Copy and complete the table using Star’s equation.

<table>
<thead>
<tr>
<th>(x) seconds</th>
<th>(y) meters run</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

b. Four hundred meters is approximately a quarter of a mile. About how long would it take Star to run one-quarter of a mile? Is this reasonable? Why or why not?

c. Star decides she is going to run for one hour. How many seconds is this?

d. According to her equation, how many meters would she run in one hour?

e. How many miles is this? 1 mile ≈ 1,600 meters

f. Is this answer reasonable? Why or why not?

18. During the summer, Jorge works at a kids camp. He was given $100 for signing on for the summer and then is paid an additional $35 per day of work. The linear equation that represents Jorge's total earnings is \(y = 100 + 35x\). Copy and complete the table that shows Jorge's total earnings based on how many days he works.

<table>
<thead>
<tr>
<th>(x) days</th>
<th>(y) earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

**REVIEW**

Solve each equation. Check your solution.

19. \(2x + 7 = 22\)

20. \(\frac{x}{5} - 9 = -3\)

21. \(-x + 2 = 8\)

22. \(4(x + 7) = 12\)

23. \(6x + 1 = 5x + 4\)

24. \(2(3x - 2) = 38\)

25. \(2x - 5 = 5x + 28\)

26. \(6 + \frac{x}{3} = 2\)

27. \(8 = 23 - 5x\)

**Tic-Tac-Toe ~ Writing Equations From Tables**

Creating equations from input-output tables is a difficult process. Create a worksheet that steps a student through the process of finding the rate of change, the start value and then writing the equation. Include tables that have a start value listed in the table and some that do not. Turn in a blank copy of the worksheet and an answer key.
Up to this point you have looked at linear relationships by examining their rates of change and start values which are also called $y$-intercepts. The rate of change tells you how much the $y$-value should increase or decrease as the counter ($x$) increases.

Rate of change is also known as slope. Like rate of change, slope is used to describe the steepness of a line. Slope is the ratio of the vertical change (the rise) to the horizontal change (the run). The easiest way to calculate the slope of a line when it is graphed is to create a slope triangle. A slope triangle is formed by drawing a right triangle where one leg of the triangle represents the vertical rise and the other leg is the horizontal run. The hypotenuse of the triangle (the longest side) is part of the line itself. Start at the point furthest to the left and go up or down to draw your first leg. Then draw your second leg to the right.

A line can have a slope that is positive, negative, zero or undefined.

**Finding Slope**

The slope of a line is the ratio of the change in $y$-values to the change in $x$-values.

$$\text{Slope} = \frac{\text{Change in } y\text{-values}}{\text{Change in } x\text{-values}} = \frac{\text{rise}}{\text{run}}$$
EXAMPLE 1

Draw a slope triangle for each line (when possible) and identify the slope of the line.

SOLUTIONS

a.

\[ \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-1}{+2} = -\frac{1}{2} \]

b.

\[ \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{+2}{+2} = 1 \]

c.

\[ \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{+3}{0} = \text{undefined} \]

d.

\[ \text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{0}{+3} = 0 \]
There are times when a graph is not provided. You may only be given a table of values or two ordered pairs. In each of these situations, you can graph the points and then create a slope triangle to calculate the slope.

**EXAMPLE 2**

Graph the line that goes through the given points, draw a slope triangle and give the slope.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

**SOLUTIONS**

a. Graph the points.

Draw the slope triangle. Start at the point furthest to the left.

Determine the lengths of the legs of the triangle.

\[
\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{-6}{+3} = -2
\]

b. Graph the points.

Draw the slope triangle. Start at the point furthest to the left.

Determine the lengths of the legs of the triangle.

\[
\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{+3}{+2} = \frac{3}{2}
\]

When given the slope of a linear relationship you can draw a line that has the given slope using rise over run. When a slope is written without a denominator, it is mathematically correct to place a 1 in the denominator. This does not change the value of the slope. For example:

\[
3 \rightarrow \frac{3}{1}
\]

**EXAMPLE 3**

Graph a line with a slope of 2.

**SOLUTION**

No y-intercept was given so put a point anywhere on the graph.

The slope of 2 can be written as \(
\frac{2}{1}
\).

From the point, rise +2 and run +1.
Find the slope of each line.

1. 
2. 
3. 
4. 
5. 
6. 

Use each table or graph to determine if the slope of the line is positive, negative, zero or undefined.

7. 
8. 
9. 

10. 
11. 
12. 

13. Karissa and Rider both calculated the slope of the same line. Rider says the slope is \(-\frac{2}{3}\) and Karissa believes the slope is \(\frac{2}{3}\).
   a. On a coordinate plane, graph a line that goes through the origin and has a slope of \(-\frac{2}{3}\).
   b. On a different coordinate plane, graph a line that goes through the origin and has a slope of \(\frac{2}{3}\).
   c. What do you notice about the two lines? The teacher says the slope is \(-\frac{2}{3}\). Is this the same slope as found by Karissa, Rider or both?
On a coordinate plane, graph a line with the given slope.

14. slope = $\frac{3}{4}$  
15. slope = $-3$  
16. slope = $0$

Draw each line through the given point that has the given slope on a coordinate plane. Name one other ordered pair that is on the line.

17. $(1, 3)$ slope = $-2$  
18. $(-2, 1)$, slope = $-\frac{2}{5}$  
19. $(3, -4)$, slope = undefined

20. Barry is building a staircase from the first floor to the second floor. The height between the two floors is 12 feet. He wants the slope of the stairs to be $\frac{4}{3}$. What is the horizontal distance that the stairs will cover?

21. A wheelchair ramp is being designed for the library entrance. The pavement pouring company advises that the slope of the ramp be $\frac{2}{7}$. If the entrance to the library is 6 feet above ground, how long will the ramp need to be?

22. When finding the slope of a line on a graph, can you choose any two points on the line? Prove the answer by determining the slope of the line shown at the right using three different slope triangles.

**REVIEW**

Evaluate each expression using the order of operations. Write all answers in simplest form.

23. \( \frac{7 - 3}{12 - 6} \)  
24. \( \frac{-1 - 8}{6 - 3} \)  
25. \( \frac{4 - (-5)}{4 - 3} \)

26. \( \frac{8 - 8}{1 - 7} \)  
27. \( \frac{7 - 2}{5 - 5} \)  
28. \( \frac{-3 - (-2)}{7 - (-10)} \)

**Tic-Tac-Toe ~ Children's Story**

Sequences occur in many real-world situations. Create a children's book that incorporates the concept of recursive sequences and recursive routines. The character(s) in your book should encounter sequences in a variety of real-world situations. The plot should include the character(s) needing to find the start values, operations and specific term in recursive sequences. Your book should have a cover, illustrations and a story line that is appropriate for children.
EXPLORING!

Ginger got a job in downtown Portland. She bought a parking pass at a garage not far from her place of work. The table shows her total parking expenses based on the number of weeks she has been parking at the garage.

<table>
<thead>
<tr>
<th>Week</th>
<th>Total Expenses, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>$50</td>
</tr>
<tr>
<td>10</td>
<td>$74</td>
</tr>
<tr>
<td>12</td>
<td>$86</td>
</tr>
<tr>
<td>24</td>
<td>$158</td>
</tr>
</tbody>
</table>

Step 1: Calculate the rate of change (the change in $y$ over the change in $x$) for the table above.

Step 2: Graph the ordered pairs on a Quadrant I coordinate plane like the one shown below. Draw a line through the points.

Step 3: Make a slope triangle and determine the slope of the line.

Step 4: What do you notice about the rate of change and the slope of the line?

Step 5: If you were given the table of values in the table at right, what would the rate of change (or slope) ratio look like?

Step 6: The ratio developed in Step 5 is called the “Slope Formula”. The subscripts identify two different points. Try your formula on these points from the table above: (6, 50) and (12, 86). Did you get the same slope as you did in Steps 1 and 3?

Step 7: You have learned three methods for finding slope: rate of change, slope triangles and the slope formula. Which method do you like the best? Why?
Subscripts designate which point you are using in your calculation. You read \( x_1 \) as “\( x \) sub one”. Think of it as saying “the \( x \)-coordinate of the first point”.

**EXAMPLE 1**
Use the slope formula to find the slope of each line that passes through the given points.

a. \((3, 2)\) and \((8, 5)\)
b. \((1, −1)\) and \((3, −5)\)
c. \((6, −2)\) and \((6, 4)\)

**Solutions**

a. Let \((3, 2)\) be \((x_1, y_1)\) and \((8, 5)\) be \((x_2, y_2)\).
Substitute the numbers into the slope formula.

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{8 - 3} = \frac{3}{5}
\]

b. Let \((1, −1)\) be \((x_1, y_1)\) and \((3, −5)\) be \((x_2, y_2)\).
Substitute the numbers into the slope formula.

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - (-1)}{3 - 1} = \frac{-4}{2} = -2
\]

It is impossible to divide by 0 so the slope is undefined.

c. Let \((6, −2)\) be \((x_1, y_1)\) and \((6, 4)\) be \((x_2, y_2)\). 
Substitute the numbers into the slope formula.

\[
\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-2)}{6 - 6} = \frac{6}{0} = \text{undefined}
\]

You have learned three methods for calculating slope. All three methods will work in any situation. Depending on the way the linear relationship is presented, there may be one method that is easier than the other two methods.

**Rate of Change** Easiest method when information is presented in an input-output table.

**Slope Triangle** Easiest method when information is presented in a graph.

**Slope Formula** Easiest method when given two ordered pairs.
**EXERCISES**

Find the slope of the line that passes through the given points.

1. (4, 7) and (6, 10)  
2. (0, 8) and (3, 5)  
3. (−1, 11) and (4, 11)  
4. (−6, −1) and (2, 0)  
5. (7, −2) and (7, 9)  
6. (6, 4) and (2, 10)  
7. (−2, 10) and (3, 10)  
8. (0, −4) and (5, 0)  
9. (9, 8) and (4, 18)  

10. In each part below, explain what method you would choose to calculate the slope. Then find the slope.

   a.  
   b. A line through (1, 5) and (−2, 9)  
   c.  

11. Consider the line through the points (3, 8) and (8, 12).

    a. Find the slope.  
    b. Convert your slope fraction to a decimal. Remember that this is your rate of change.  
    c. The rate of change tells you how much to increase or decrease for every one-unit step. Copy and complete the table using the rate of change.  
    d. How many ordered pairs for this line do you have now? How many more could you figure out?  

12. Tanika joined a gym. At 4 months, she had paid a total of $94 in membership fees. After 9 months, she had paid a total of $204 in membership fees. Let $x$ represent the number of months she has been a member and $y$ represent the total she has paid in membership fees.

    a. Write two ordered pairs to represent Tanika’s gym membership information.  
    b. Find the slope of the line that contains the two points in part a.  
    c. The slope represents the rate of change in real-world situations. What does the slope represent in terms of Tanika’s gym membership fee?  

13. Scott skis at Willamette Pass during the winter. His favorite run has a vertical descent of 1,560 feet. The run covers a horizontal distance of 3,900 feet. What is the slope of this particular run? Give the answer as a fraction and as a decimal.  

14. Devin drove up a hill. After he drove 4 minutes, he was at an elevation of 620 feet. After he drove 12 minutes he was 1,580 feet high.

    a. Should the $x$-values represent minutes or feet? Why?  
    b. Calculate the rate of change in this situation. What method did you use and why?  
    c. If he continues to climb at this rate, how much elevation will he gain in the next 15 minutes?
15. Nigel put a two-liter bottle of soda in his locker. He did not realize there was a hole in the bottom of the container and that the liquid had slowly dripped out. The graph represents the amount of soda left in the bottle based on the amount of time that had passed.
   a. What is the slope of the line?
   b. What does the value of the slope represent in this situation?
   c. How many hours had passed when Nigel’s soda bottle became empty?

**REVIEW**

Match each recursive rule with its linear equation.

16. Start Value: 4   Slope: \( \frac{1}{2} \)   A. \( y = 3x + 1 \)

17. Start Value: \(-2\)   Rate of Change: \( \frac{1}{2} \)   B. \( y = \frac{1}{2} - 2x \)

18. \( y \)-Intercept: 1   Slope: 3   C. \( y = 2x \)

19. Start Value: \( \frac{1}{2} \)   Rate of Change: \(-2\)   D. \( y = x + 3 \)

20. \( y \)-Intercept: 3   Rate of Change: +1  E. \( y = 4 + \frac{1}{2}x \)

21. Start Value: \(-2\)   Slope: 0  F. \( y = \frac{1}{2}x - 2 \)

22. \( y \)-Intercept: 0   Rate of Change: +2  G. \( y = -2 \)

**Tic-Tac-Toe ~ Slope Methods**

You have learned to find the slope of a line when given either a graph, table or two ordered pairs. Create a flip book that explains how to use the slope formula, slope triangles and input-output tables to find slope. Include examples and diagrams.
Lesson 8 ~ Recursive Routines

Copy each sequence of numbers and fill in the missing values. Identify the start value and the operation that must be performed to arrive at the next term.

1. 8, 1, −6, ____, ____, ____
2. 92, 110, ____, 146, ____, ____
3. 7, 7.6, ____, ____ 9.4, ____
4. ____, ____, 19, 22, 25, ____

For each sequence below, describe the recursive routine (start value and operation) and give the 9th term in the sequence.

5. 18, 5, −8, −21, …
6. \(\frac{2}{5}, \frac{4}{5}, 1\frac{1}{5}, 1\frac{3}{5}\), …

7. Draw the next two figures in the following pattern. Each block is 1 unit by 1 unit.

   - a. What is the perimeter of the first figure?
   - b. What is the perimeter of the second figure? The third figure?
   - c. Write the recursive routine (start value and operation) that describes the perimeters.
   - d. Predict the perimeter of the seventh figure in this pattern.
Lesson 9 ~ Linear Plots

Describe the linear relationship given by the y-coordinates on each linear plot below by stating the start value and operation. Create an input-output table showing the ordered pairs represented by each plot.

8.

9.

10. Create a linear plot for the first five ordered pairs in the given recursive routine describing the y-coordinates.
    a. Start Value: 11
       Operation: Subtract 2
    b. Start Value: − 4
       Operation: Add $\frac{1}{2}$

Lesson 10 ~ Recursive Routine Applications

11. Determine an appropriate range for the y-axis and state what increments should be used on the graph.
    a. |
        | Hours | Miles Traveled |
        |-------|----------------|
        | 0     | 55             |
        | 1     | 110            |
        | 2     | 165            |
        | 3     | 220            |
        | 4     | 275            |
    b. |
        | Years | Savings       |
        |-------|---------------|
        | 0     | $10,200       |
        | 1     | $9,400        |
        | 2     | $8,600        |
        | 3     | $7,800        |
        | 4     | $7,000        |

12. LaQuisha borrows $150 from her parents to buy school clothes. Each week she uses $12 of her allowance to repay her parents.
    a. Write a recursive routine that describes LaQuisha’s balance based on the number of weeks that have passed since she borrowed the money.
    b. Fill in an input-output table that will give her balance for 0 to 5 weeks.
    c. Create a linear plot that shows LaQuisha’s balance through the first five weeks.
    d. Use your calculator to determine how many weeks it will take before LaQuisha has repaid her parents. How much was her last payment?
13. In Oregon, the cost of a gallon of gasoline in April 2008 averaged $3.35. One analyst predicted that the cost of gas would rise $0.40 per year.
   a. Write a recursive routine that describes the price of gasoline based on the number of years that have passed since April 2008.
   b. Create an input-output table for the value of a gallon of gas for 0 to 5 years. Let 2008 represent year 0.
   c. Create a linear plot that shows the cost of a gallon of gasoline through the first five years.
   d. Determine how many years it will take before the cost of gasoline will be over $10 per gallon according to this analyst’s prediction.

Lesson 11 ~ Rate of Change

Determine the rate of change for each situation.

14. Jordan collected 28 coins in 4 days.

15. Rebecca drove 270 miles in 6 hours.


Determine the rate of change and start value for each table.

17. | x  | y  |
    |----|----|
    | 0  | 2  |
    | 1  | 9  |
    | 2  | 16 |
    | 3  | 23 |
    | 4  | 30 |

18. | x  | y  |
    |----|----|
    | 0  | 33 |
    | 3  | 0  |
    | 5  | −22|
    | 6  | −33|
    | 8  | −55|

19. | x  | y  |
    |----|----|
    | −1 | 9  |
    | 1  | 17 |
    | 3  | 25 |
    | 5  | 33 |
    | 8  | 45 |

20. Maria began an exercise plan at the beginning of the school year. In the table shown below, Maria records her weight at different points during the school year. Assume Maria loses the same amount every week.
   a. How many pounds is Maria losing each week?
   b. How much did she weigh when she first started this exercise plan?
   c. If this pattern continues, how much will she weigh 17 weeks into her exercise plan?
   d. Does it make sense that this pattern will continue throughout the whole school year? Why or why not?
Lesson 12 ~ Recursive Routines to Equations

Write the linear equation for each recursive rule.

21. Rate of Change = +6  
   Start Value = −2
   \( y \)-Intercept = 6

22. Rate of Change = \(-\frac{1}{2}\)
   Start Value = 1

Determine the rate of change and \( y \)-intercept for each table. Write a linear equation that represents each table.

24.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>39</td>
</tr>
<tr>
<td>1</td>
<td>33</td>
</tr>
<tr>
<td>2</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
</tbody>
</table>

25.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td>2.9</td>
</tr>
<tr>
<td>−1</td>
<td>6.4</td>
</tr>
<tr>
<td>0</td>
<td>9.9</td>
</tr>
<tr>
<td>1</td>
<td>13.4</td>
</tr>
<tr>
<td>3</td>
<td>20.4</td>
</tr>
</tbody>
</table>

26.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−2</td>
<td>−15</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>13</td>
</tr>
<tr>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>10</td>
<td>33</td>
</tr>
</tbody>
</table>

27. Madison receives $42 at the beginning of the month to use for lunch money. Each day at school she buys the lunch special and a milk. This costs her $3.10. Madison wants to develop a linear equation to calculate how much money she has left based on the number of days she has bought lunch during a month.
   a. What is the \( y \)-intercept in this situation?
   b. What is the rate of change?
   c. Write a linear equation to represent the amount of lunch money Madison has left based on the number of days this month she has bought lunch.

Lesson 13 ~ Input-Output Tables from Equations

Determine the rate of change and the \( y \)-intercept from the given equations.

28. \( y = 5x + 1 \)

29. \( y = 7 - 6x \)

30. \( y = 2 + \frac{1}{2}x \)

31. \( y = 2x + 7 \)

32. \( y = \frac{2}{3}x \)

33. \( y = 8 \)

Given the equation, copy and complete the input-output tables.

34. \( y = 3x + 1 \)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

35. \( y = \frac{1}{3}x + 4 \)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>

36. \( y = 8x + 7 \)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>−7</td>
<td></td>
</tr>
<tr>
<td>−3</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
</tr>
</tbody>
</table>
Lesson 14 ~ Calculating Slope from Graphs

Find the slope of each line.

37. [Graph]

38. [Graph]

39. [Graph]

Create a coordinate plane, draw a line through the given point that has the given slope. Name one other ordered pair that is on the line.

40. (0, 0), slope = $\frac{1}{2}$

41. (−1, 3), slope = −4

42. (0, 2), slope = 0

43. A ladder leaned against a wall at a slope of $\frac{4}{3}$. The top of the ladder was 12 feet off the ground. How far is the bottom of the ladder from the base of the wall?

44. The ladder in Exercise 43 is leaning against another wall only 10 feet off the ground. The bottom of the ladder is 10 feet away from the base of the building.
   a. What is the slope of the ladder in this position?
   b. Is it steeper than the slope of the ladder in Exercise 43? Why or why not?

Lesson 15 ~ The Slope Formula

Find the slope of the line that passes through the given points.

45. (3, 2) and (6, 6)

46. (0, 5) and (3, 8)

47. (−2, 9) and (8, 9)

48. (3, −2) and (5, −5)

49. (3, 2) and (3, 4)

50. (7, 5) and (3, 11)

51. For each part below, explain which method you would choose to calculate the slope. Then find the slope.
   a. | x | y |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>21</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
</tr>
</tbody>
</table>
   b. A line through (3, 6) and (−1, 11)
   c. [Graph]
Angelina’s family planned to rent a vacation home in Sunriver, Oregon. For 3 nights it would cost a total of $510. For 5 nights it would cost a total of $800. Let \( x \) represent the number of nights the house would be rented and \( y \) represent the total cost.

a. Write two ordered pairs to represent the house rental information.

b. Find the slope of the line that contains the two points in part a.

c. The slope represents the rate of change in real-world situations. What does your slope represent in terms of Angelina’s house rental?

---

**Tic-Tac-Toe ~ Similar Slope Triangles**

Use graph paper to draw three coordinate planes each from −10 to 10 on both axes.

Step 1: On the first coordinate plane, draw a line with a slope of \( \frac{2}{3} \).

Step 2: Draw three different sizes of slope triangles on the line. For example, the graph below shows three different slope triangles for a line with a slope of \( -\frac{1}{3} \).

Step 3: Write the slope fraction for each triangle.

Step 4: What do you notice about the slope fractions?

Step 5: On the second coordinate plane, draw a line with a slope of −2.

On the third coordinate plane, draw a line with a slope of \( \frac{1}{4} \).

Repeat Steps 2-4 for these two graphs.

Step 6: Can you draw any size of slope triangle on a line and get the correct slope? Are all slope triangles drawn on a given line similar? Summarize your results. Use math to support your answers.
**Karen**  
**Newspaper City Editor**  
**Albany, Oregon**

I am an editor for a daily newspaper. Our paper has a circulation of about 17,000. I coordinate local news coverage and oversee a staff of seven reporters. I also edit stories, decide when and where stories will run and design the local pages. Most of my time is spent working with the reporters on stories and laying out how each page will look. I also respond to telephone calls and emails from the public. Other duties include overseeing a monthly special section on homes and gardens and leading staff meetings twice a month.

Math is part of my job in at least a small way every day. I use ratios and percentages to figure out how articles and pictures of varying sizes fit into a given space. I approve time cards and expense reports so I need to double check the figures for correct addition and multiplication. Statistics are an important element of many news stories. I make sure numbers in the stories are correct and logical for the situation. For example, our paper may do a story on how crime has changed over the past three years. This might include overall figures, a breakdown of crime totals and percentages showing the increases or decreases from prior years. It might also compare types of crimes. The first thing I do is double check the figures to make sure the reported values correspond to the text. Secondly, if the story says overall crime is down but property crime is on the rise, the numbers we print had better show that fact.

I received a Master of Arts degree in journalism. I also have a bachelor’s degree in Biology, though most people in my profession have degrees in journalism, writing or English. Salaries depend on where you work (the size of the city) as well as experience. A typical starting salary for a beginning copy editor is around $20,000 per year. A beginning city editor might start around $30,000 per year. An executive editor earns $50,000 per year or more.

Creating a newspaper is exciting. There is the pressure of deadlines, the challenge of editing, the creativity of page design and the sense of accomplishment that comes each day when you flip through the paper that you created. There is great satisfaction creating a product every day and knowing you did your best to make it meaningful for the thousands of people it reaches.